Principles of Acoustoelectronic

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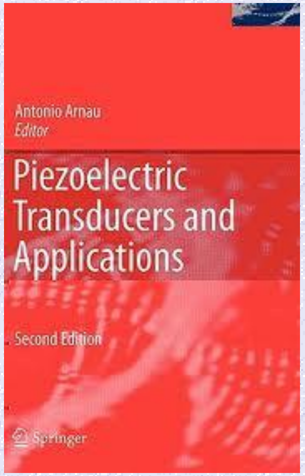
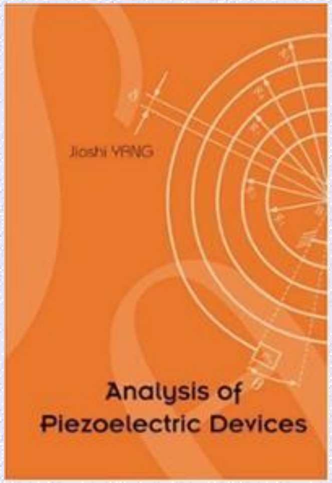
	Topics and problems	Total hours	Teaching hours
	I. Theoretical study		
1.	Basic properties of elastic vibrations and waves in a solid state.	4	1
2.	Ferroelectric materials.	4	1
3.	Elastic vibrations and waves excitation and detection.	4	1
4.	Acoustoelectronic filters and resonators.	4	1
5.	Acoustoelectronic sensors and special devices.	6	1
6.	Acoustoelectronic actuators.	4	1
	II. Laboratory – practical tests		
1.	Filters and resonators measurements	2	2
2.	Quartz crystal microbalance devices	2	2
		30	10

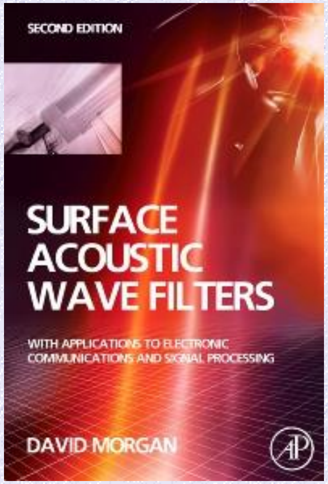
Literature

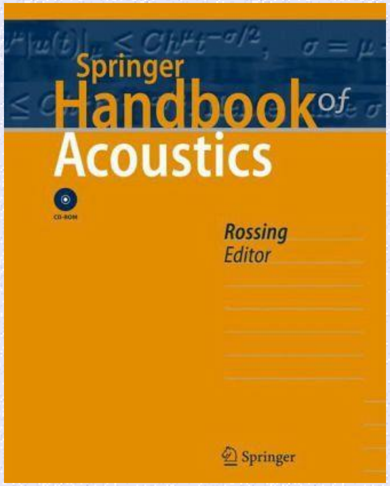
basic:
A. Arnau, Piezoelectric Transducers and Applications, Springer 2008
J. Yang, Analysis of piezoelectric devices, World Scientific Publishing 2006

additional:
D. Morgan, Surface Acoustic Wave Filters, Elsevier 2007
T. D. Rossing ed., Springer Handbook of Acoustics, Springer 2007





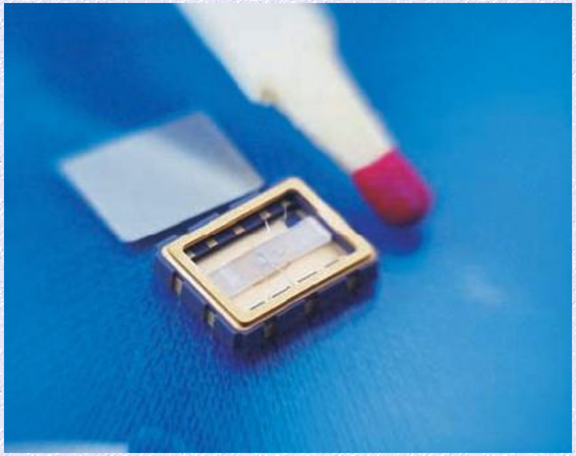




ACOUSTOELECTRONIC

Part of electronic concernings various phenomena of acoustic wave propagation inside volume of solids (bulk acoustic waves - BAW), at their surfaces (surface acoustic waves - SAW) and in different kind of waveguides as well as interactions of the acoustic waves with electric charges

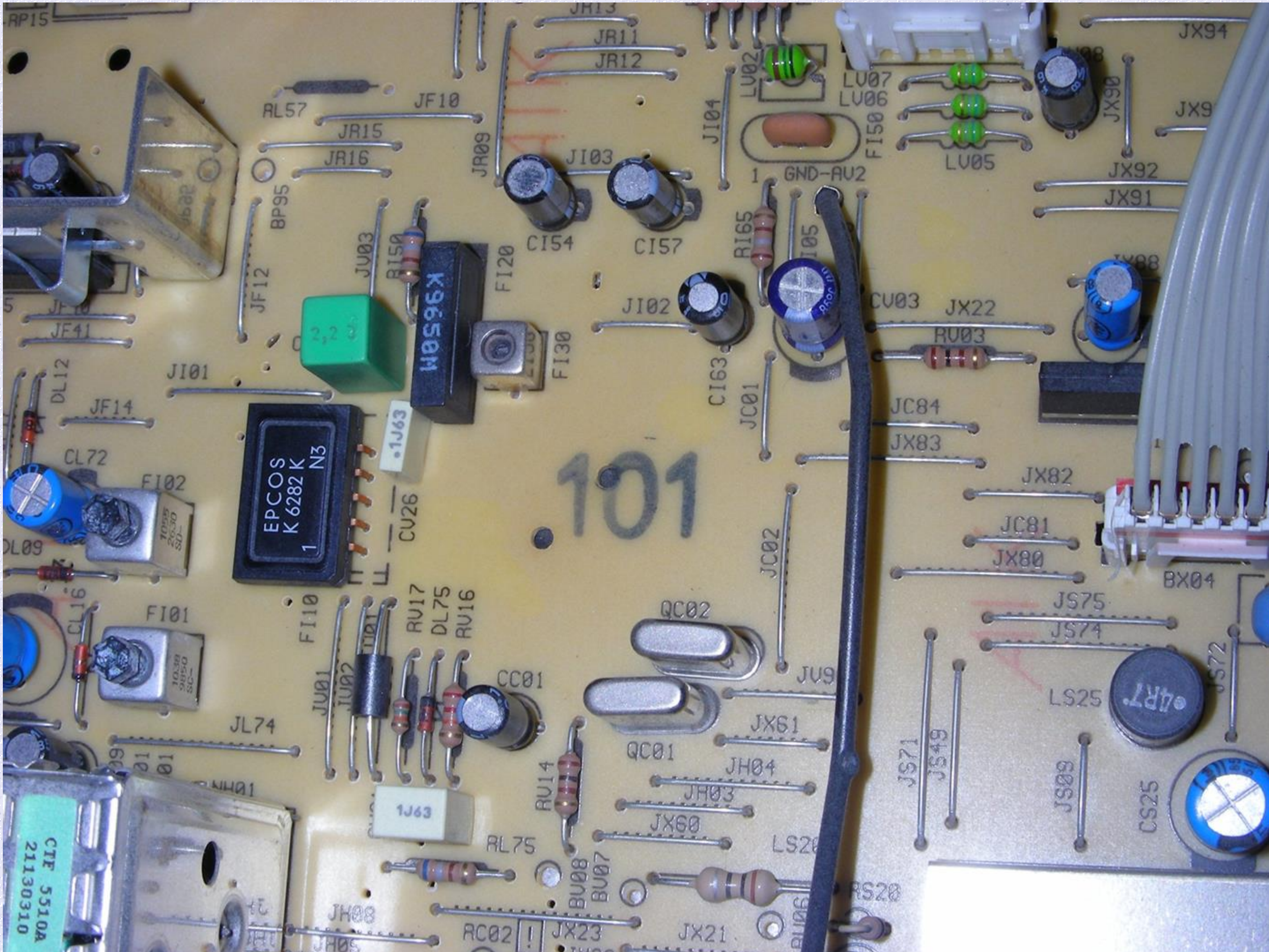
Typical acoustoelectronic devices:
filters
resonators
dispersive and nondispersive delay lines
analog signal processors
sensors
RFID devices
actuators



Advantages:
relative simplicity of design and technology
small size
high reliability
relative high operating frequencies.

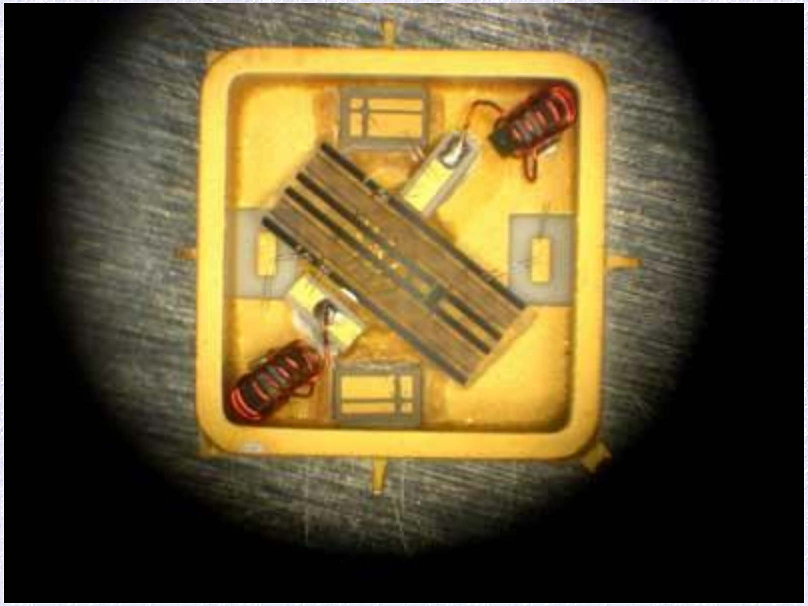
MODERN ELECTRONIC

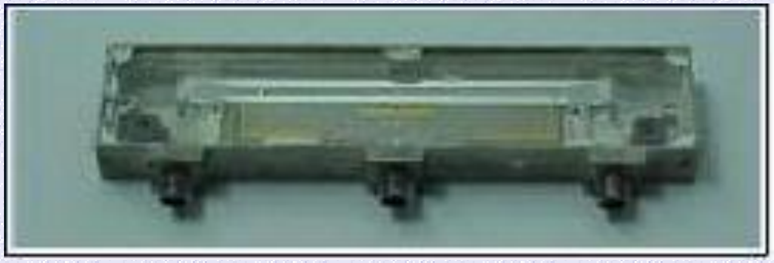
- ANALOG TECHNOLOGY
- DIGITAL TECHNOOLOGY
- MICROWAVES TECHNOLOGY
- OPTOELECTRONICS
- ACOUSTOELECTRONICS
- QUANTUM ELECTRONIC

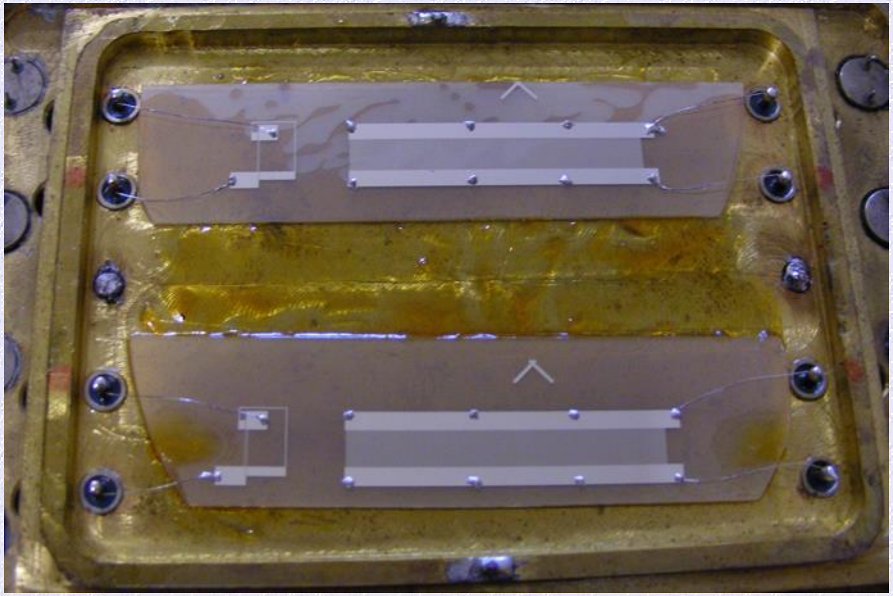


JR11
JR12
JF10
RL57
JR15
JR16
JR09
JI03
JI04
LV02
LV07
LV06
LV05
FI50
JX94
JX90
JX92
JX91
GND-AV2
BP95
JF12
JV03
RI50
K9650M
FI20
CI54
CI57
RI65
JI02
CV03
JX22
RV03
FI30
JF41
DL12
JI01
JF14
CI63
JC01
JC84
JX83
CL72
FI02
EPCOS
K 6282 K
N3
1J63
CV26
.101
JX82
JC81
JX80
BX04
DL09
JC02
JS75
JS74
FI10
RV17
DL75
RV16
FI01
QC02
JV01
JV02
CC01
LS25
4R7
JS72
JL74
JX61
QC01
JH04
JS71
JS49
JS09
RV14
JH03
CS25
1J63
JX60
RL75
BV08
BV07
JH08
RC02
JX23
JX21
RS20
CTF 5510A
21130310

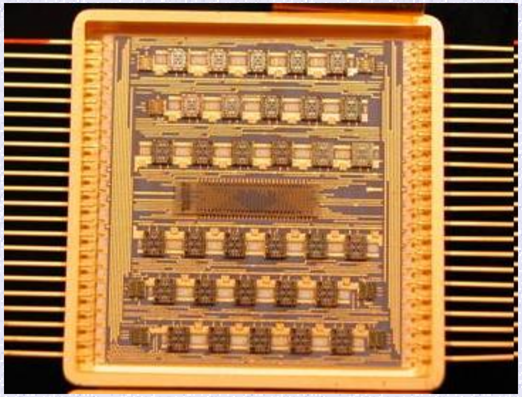
Echolocation - radars and sonars







telecommunication



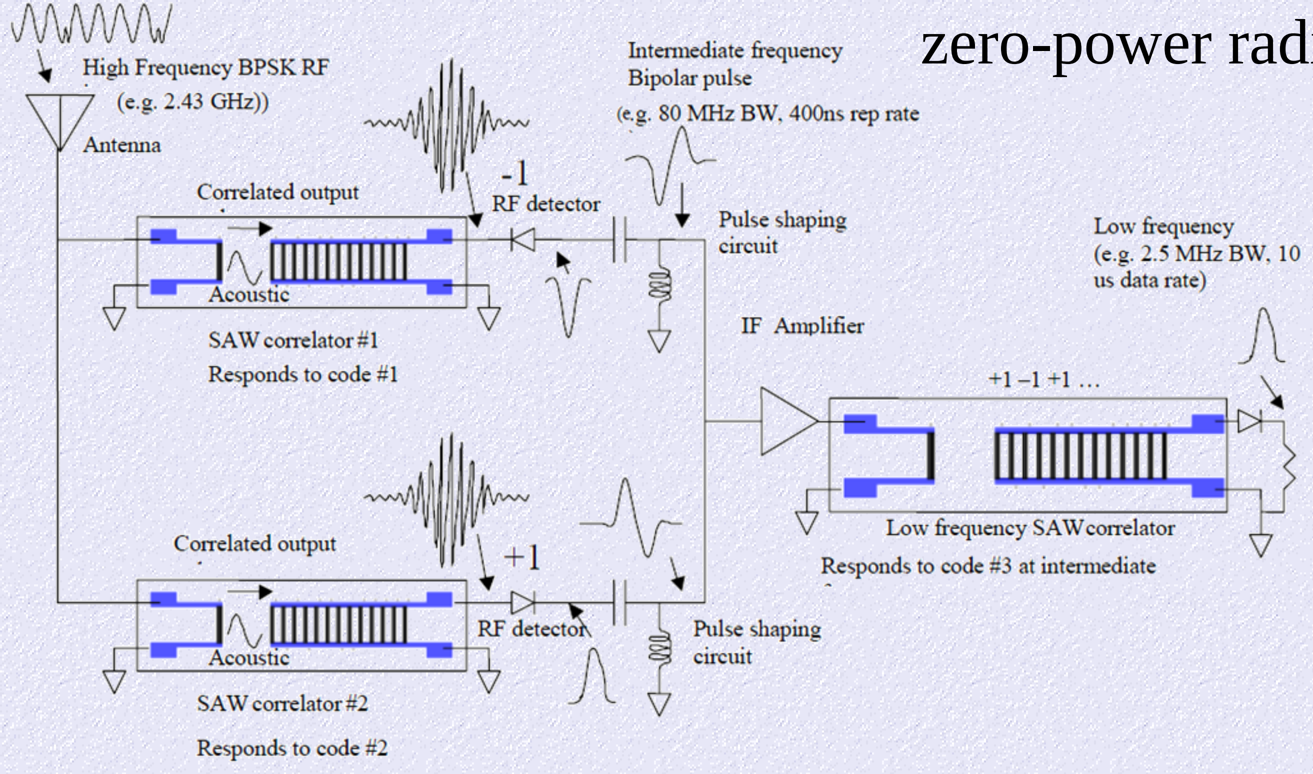
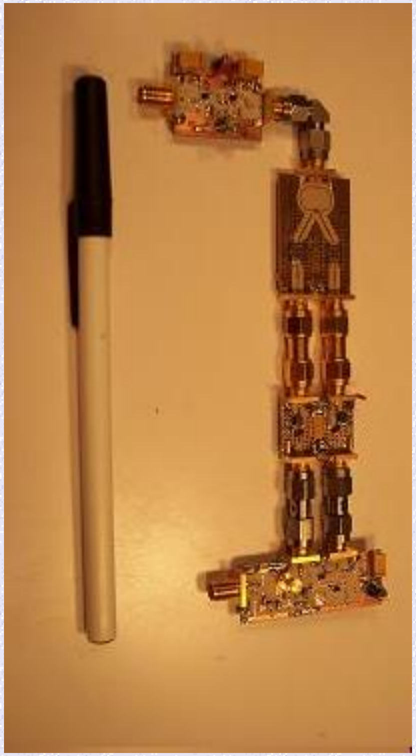
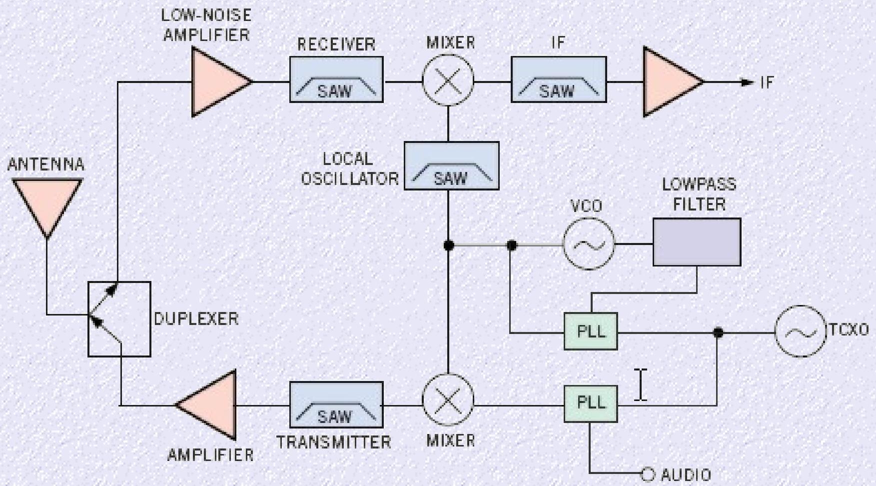


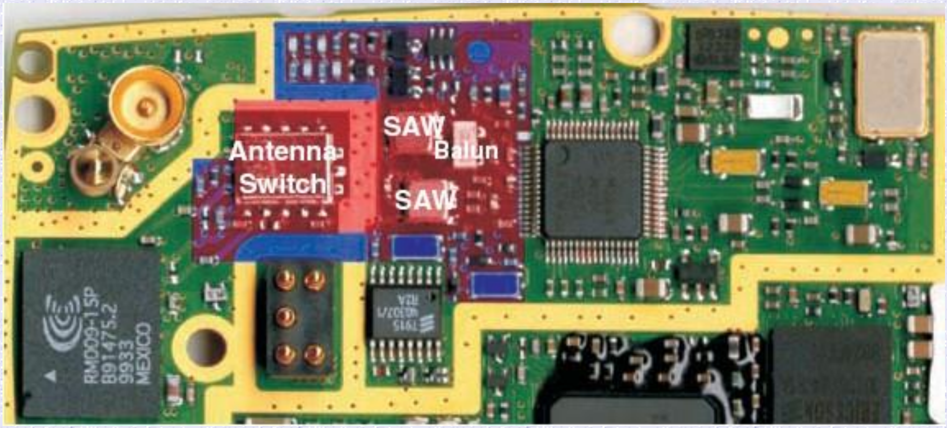
Figure 7 Very Long Code SAW Correlator

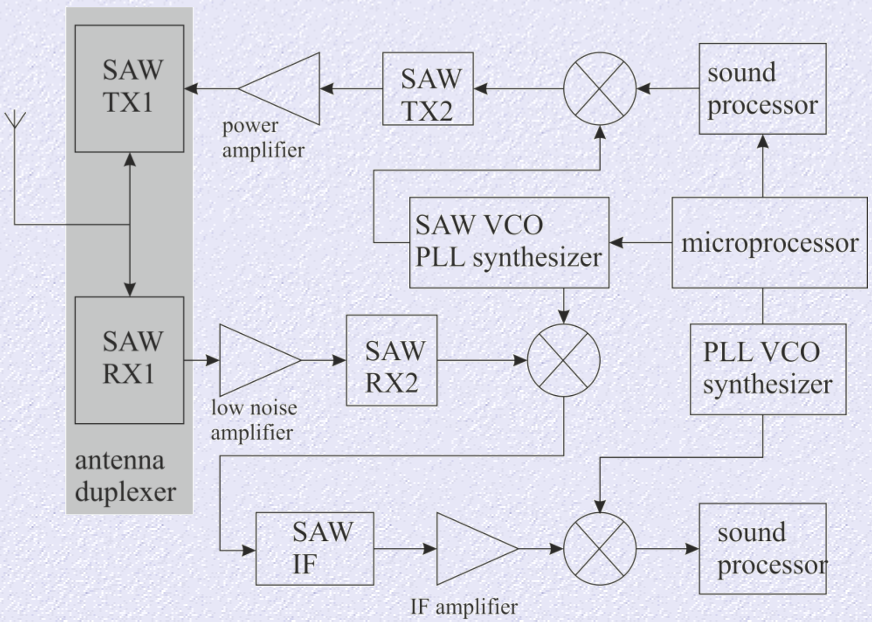
zero-power radio



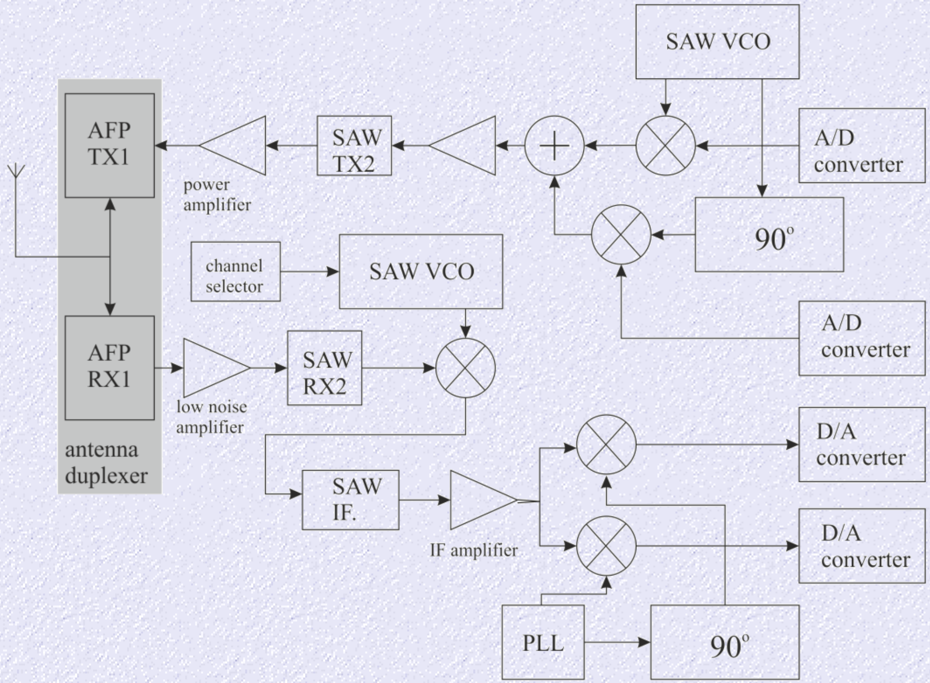
Mobile phones





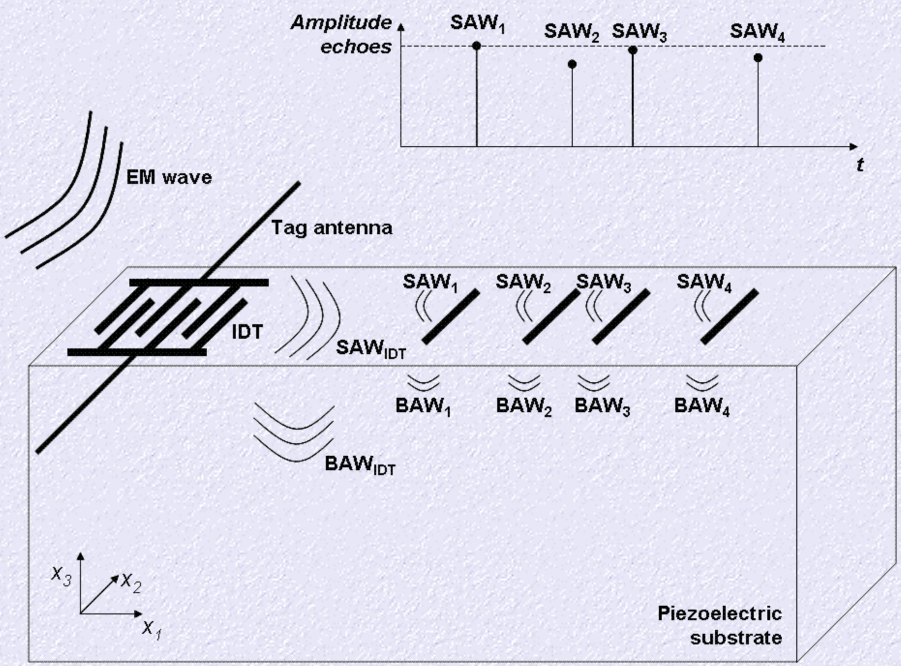


AMPS mobile phone

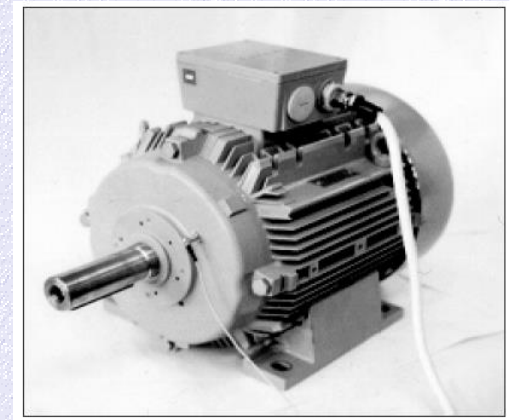


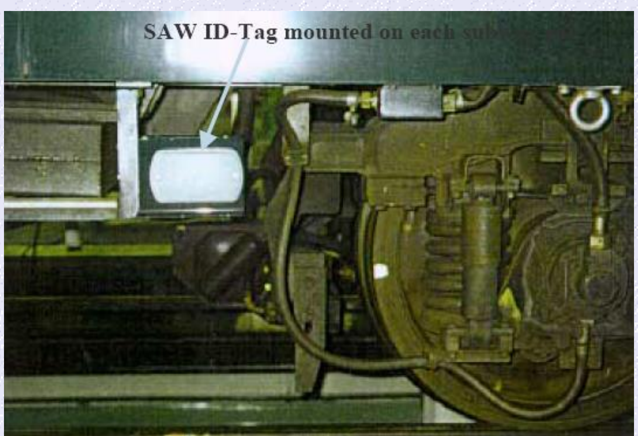
mobile phone

RFID

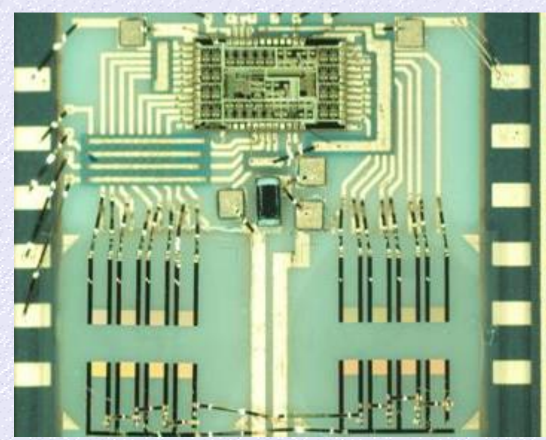


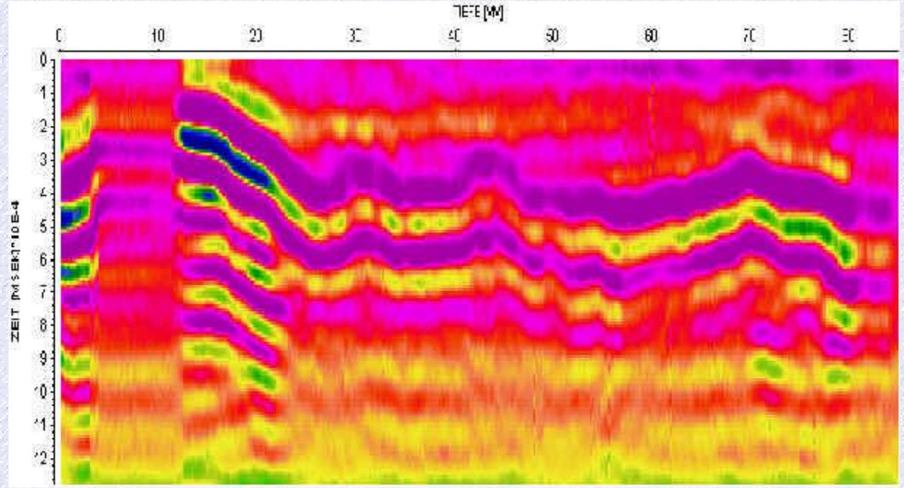
Amplitude echoes
SAW1
SAW2
SAW3
SAW4
t
EM wave
Tag antenna
IDT
SAWIDT
SAW1
SAW2
SAW3
SAW4
BAW1
BAW2
BAW3
BAW4
BAWIDT
x3
x2
x1
Piezoelectric substrate

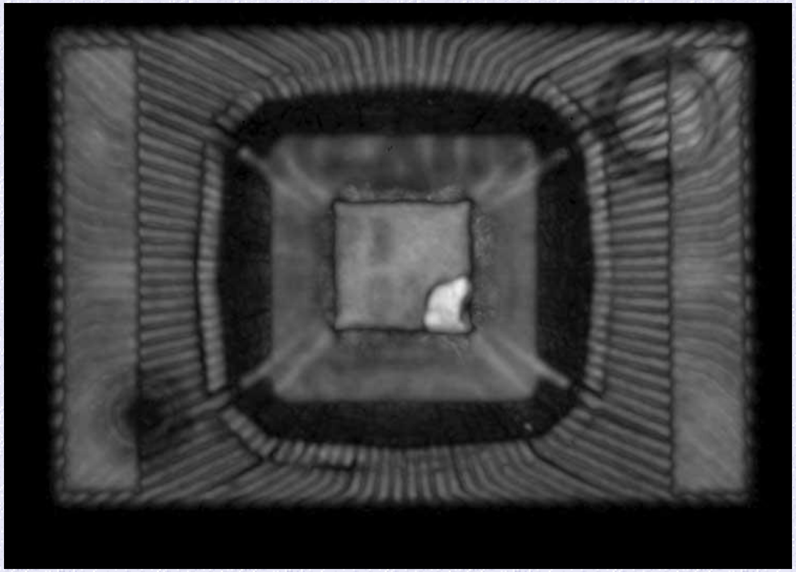


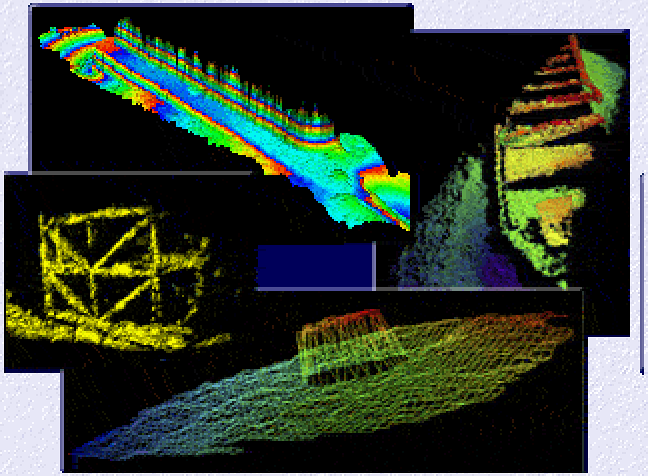


SAW ID-Tag mounted on each



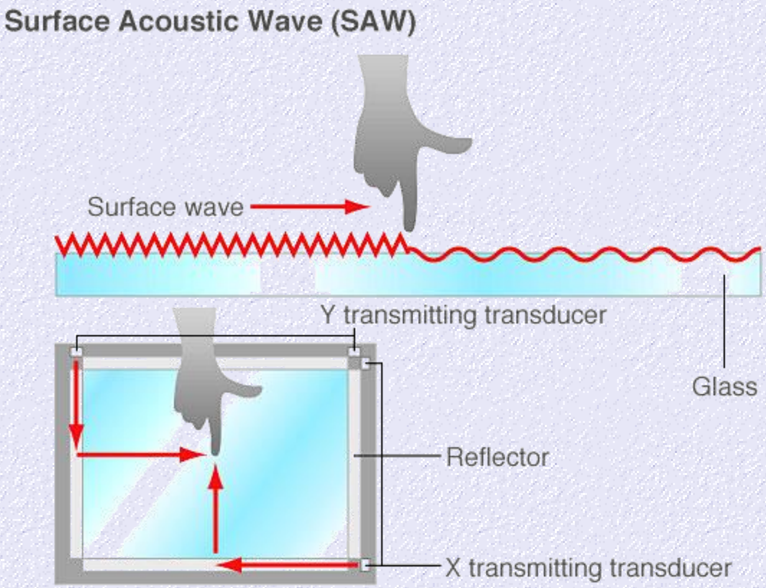




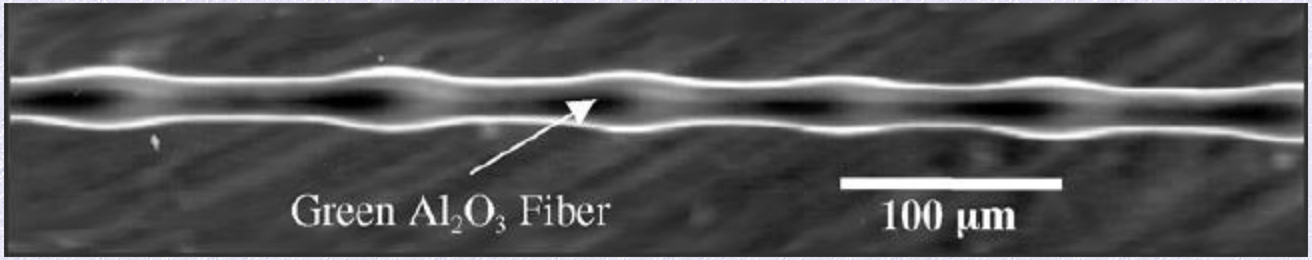


Acoustical imaging (NDT/NDE)

Touch sensitive panels



Energy harvesting systems



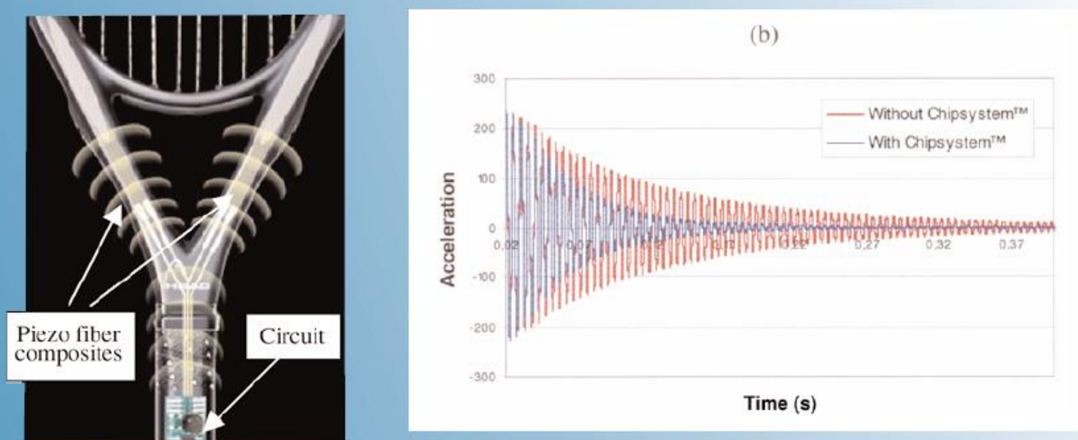


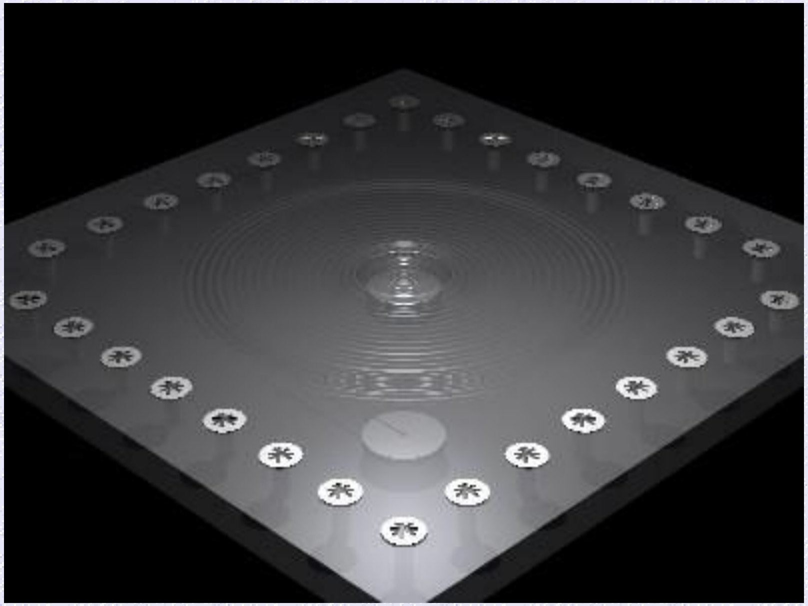
Fig. 4. (a) Drawing showing location of ACI's piezoelectric fibers in Head tennis racket. (b) Damping ringdown curves for rackets tested without and with the Chipsystem™, which consists of ACI's fiber composites coupled with an electronic circuit.

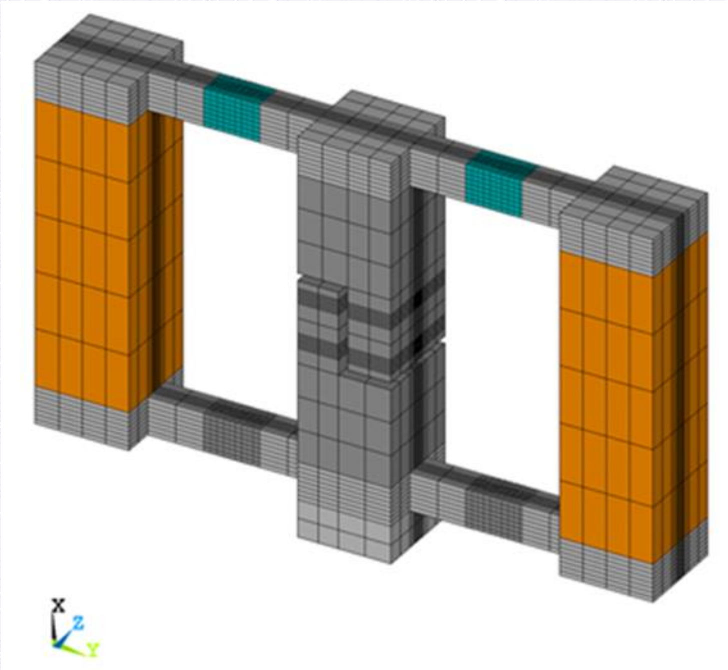
Fig. 3. Andre Agassi with ACI/Head smart tennis racket.

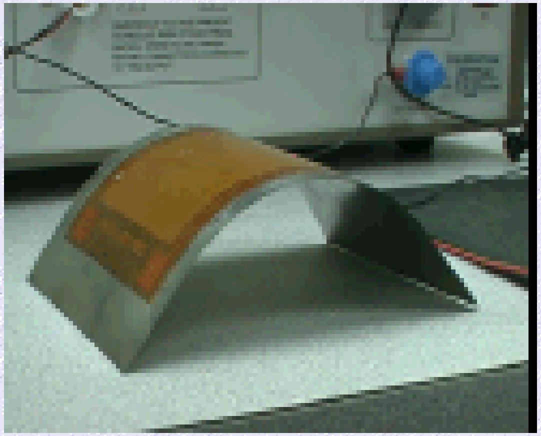
Fig. 5. ACI/Head smart ski.

smart materials

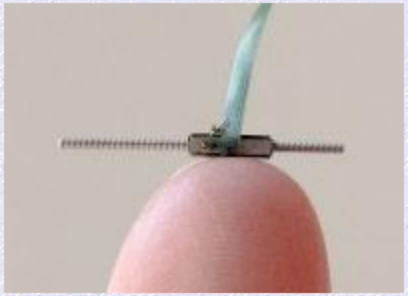
sensor + actuator

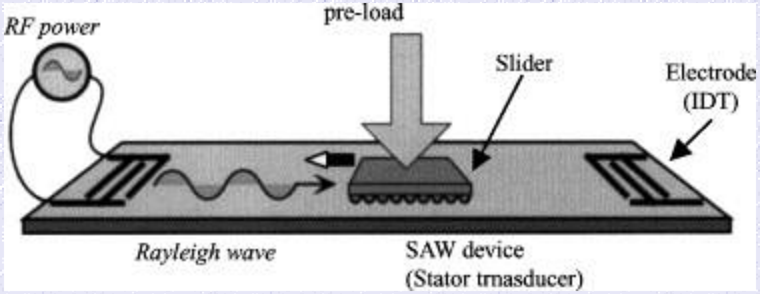


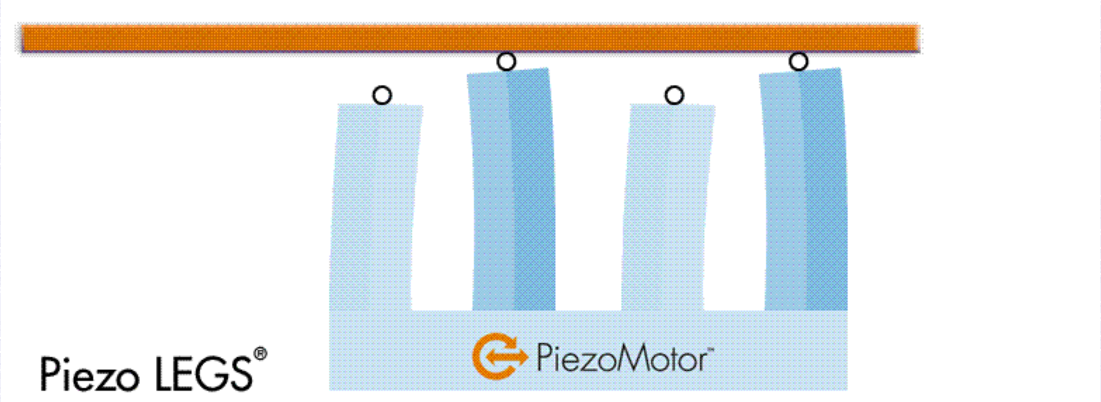


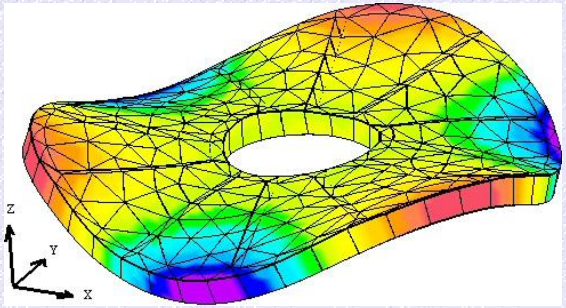


Rotary and linear motors

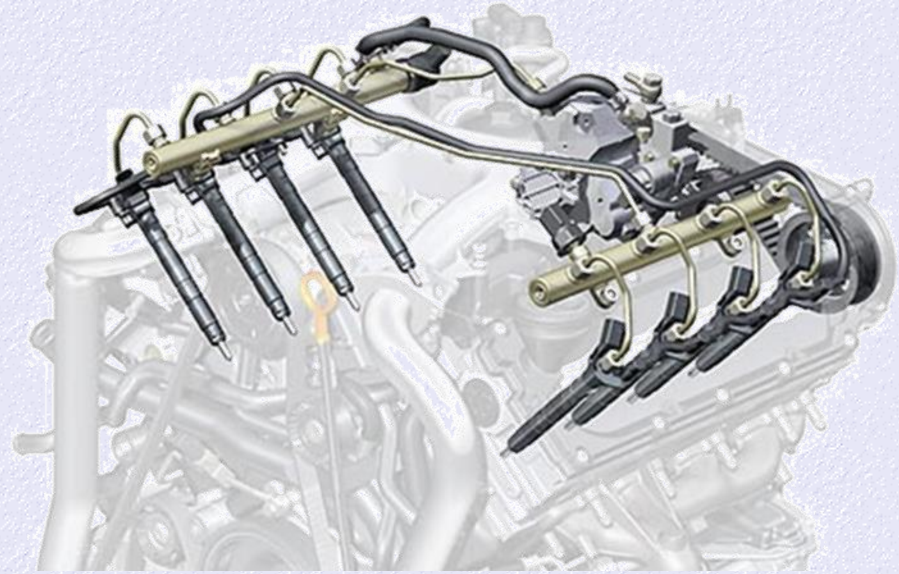


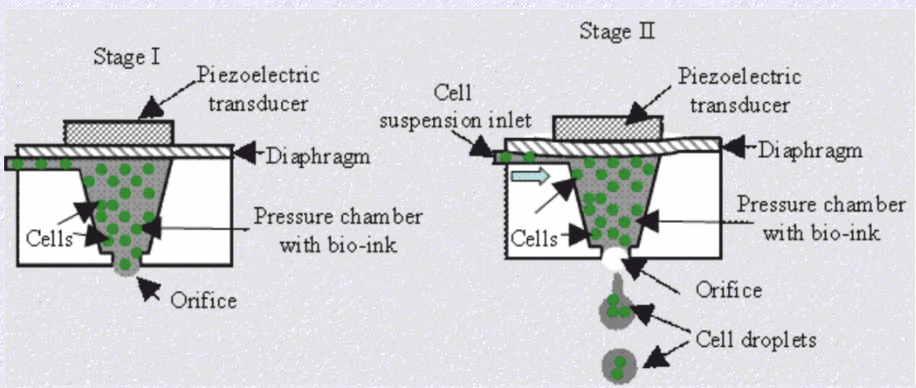


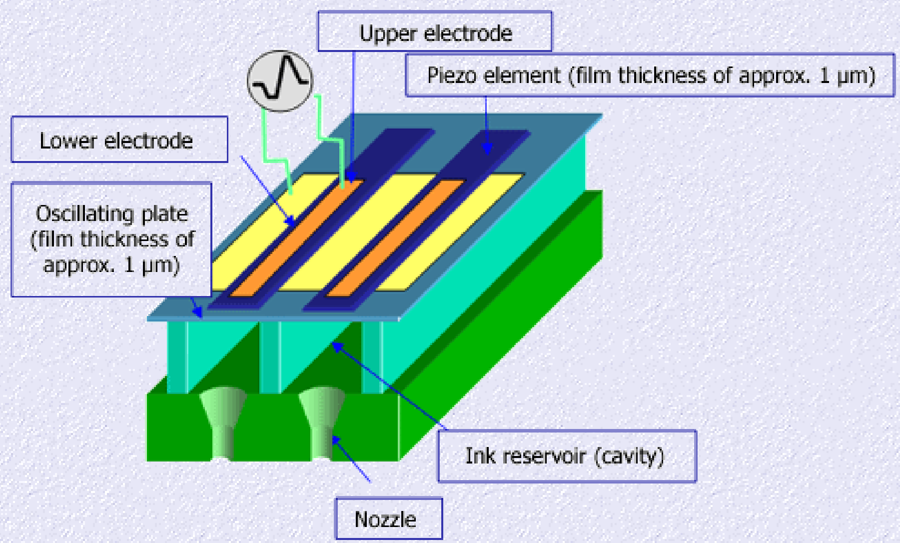




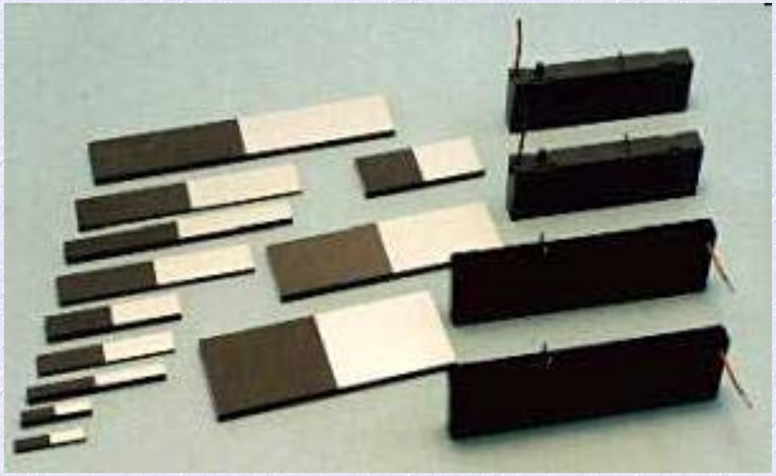
Nozzles, valves …

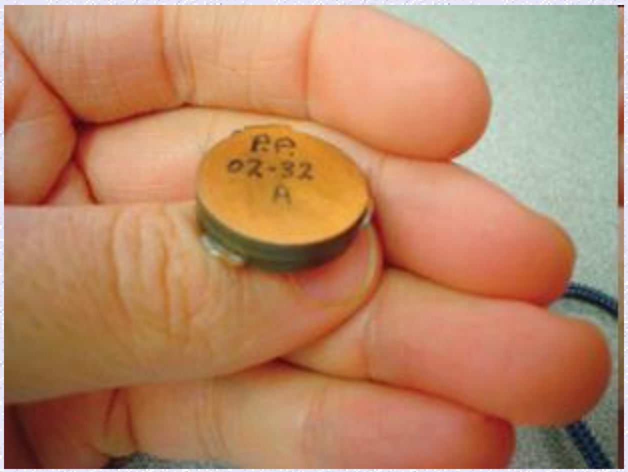


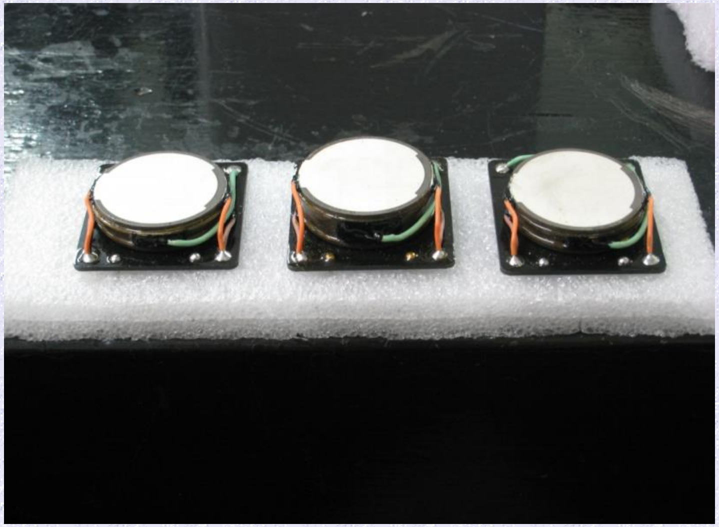


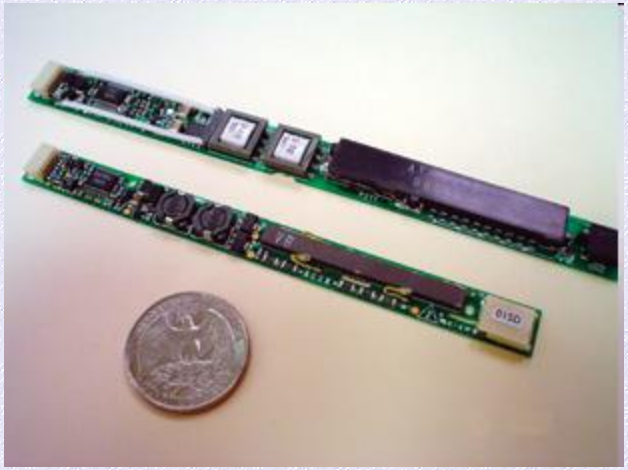


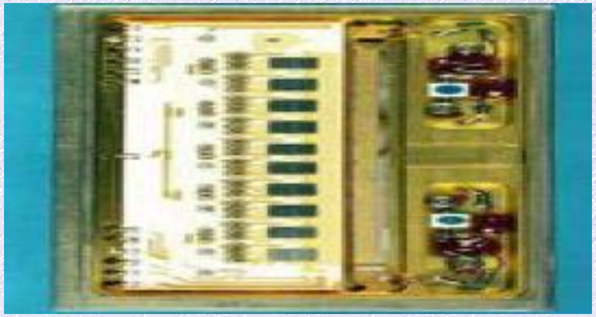
piezotransformers

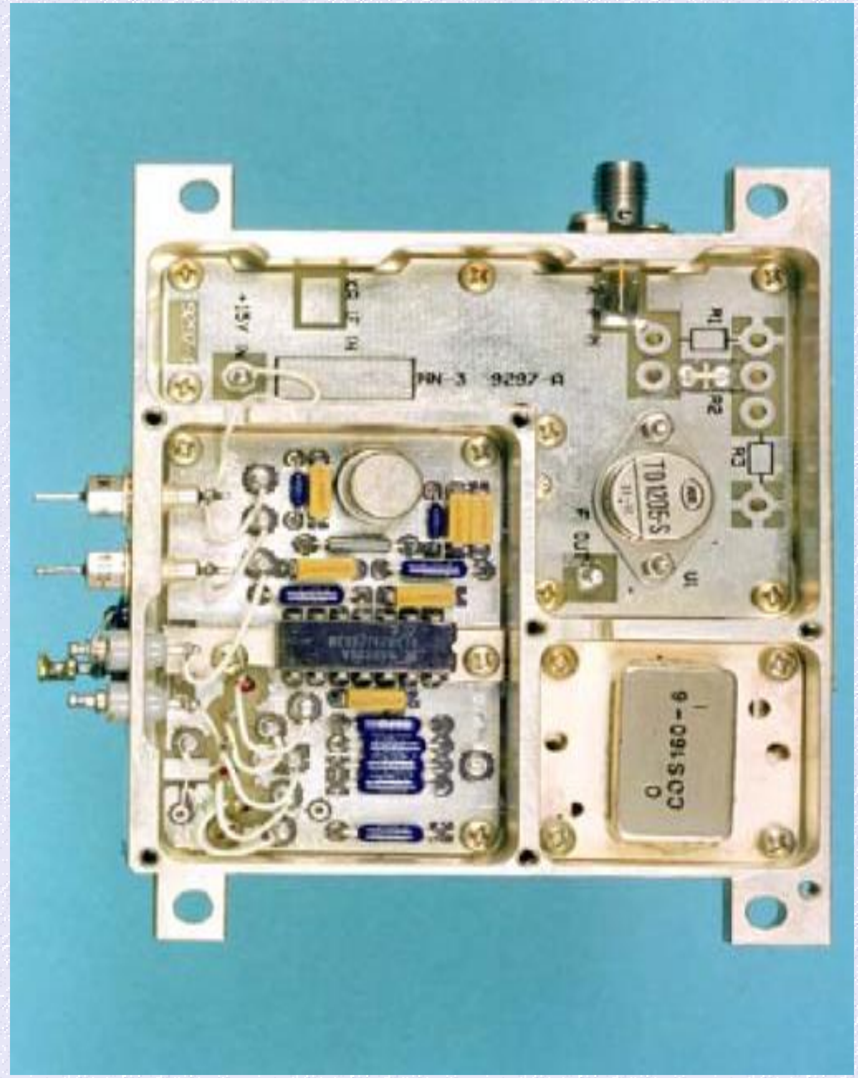








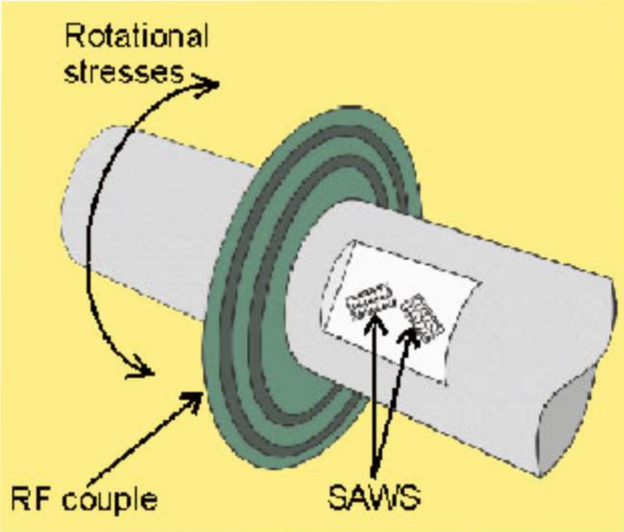


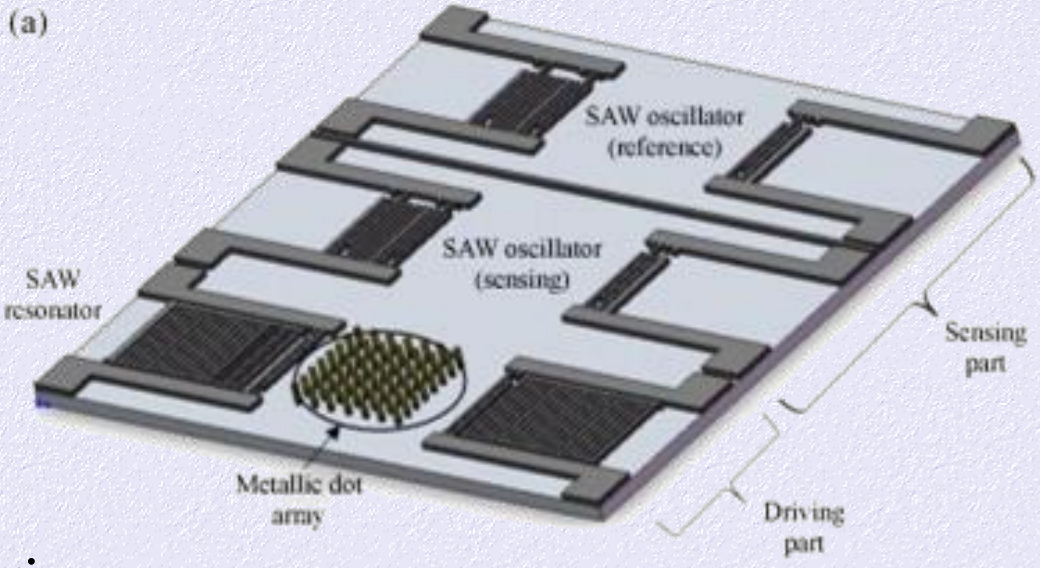


Real time signal processors

Sensors

torque

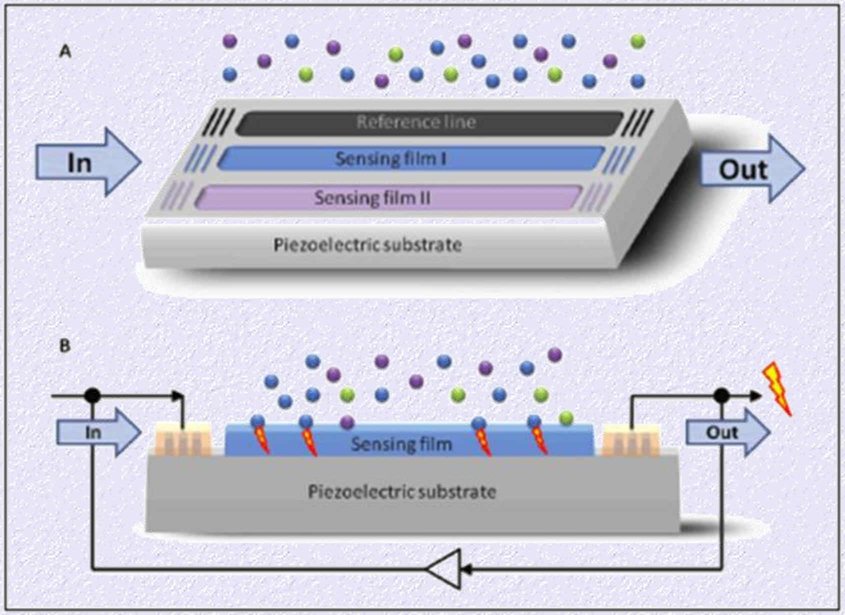


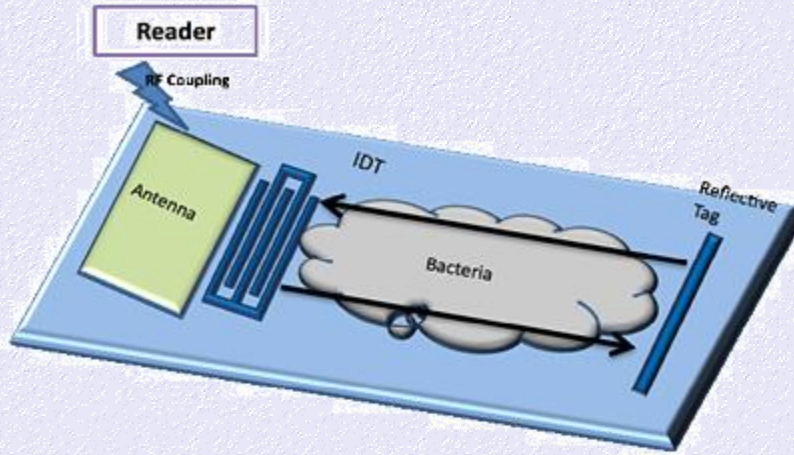


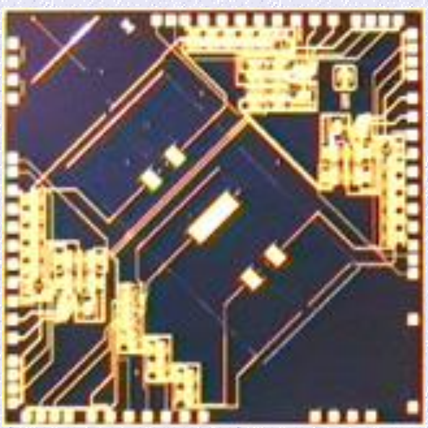
giroscope

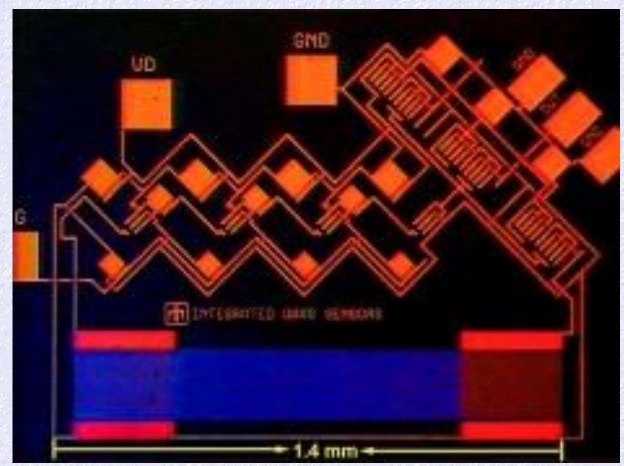
accelerometer

Chemical vapour and compounds

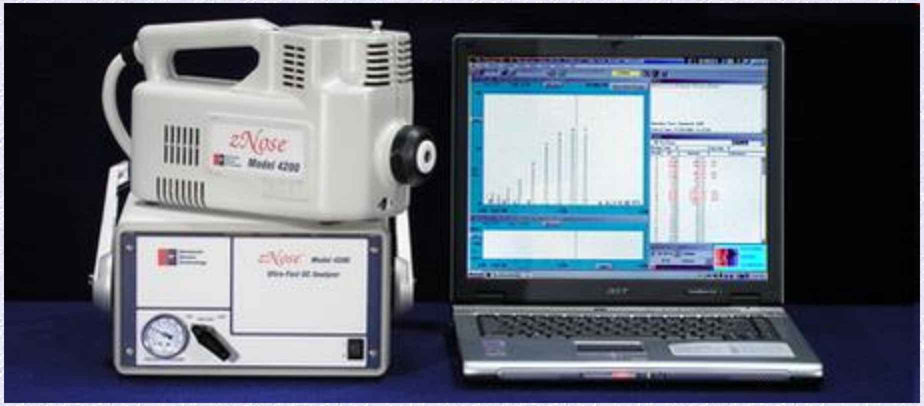


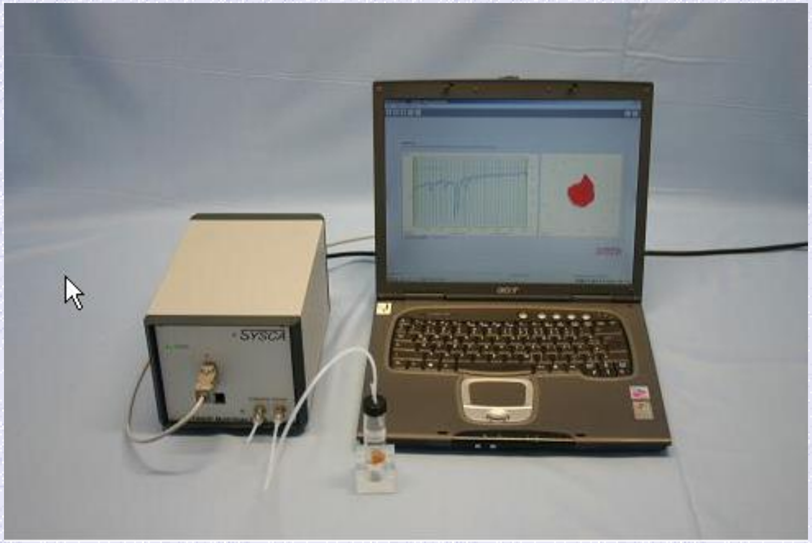






Electronic noses





Acoustic waves in solids

1D case - acoustic waves in string

Newton formula - velocity $c = \sqrt{\dfrac{T}{\rho}}$

T string tension
ρ String material density

Let us assume that the solution of 1D wave equation for string has the following form

$$u(t,x) = (A\sin kx + B\cos kx)\sin(\omega t + \varphi)$$

both ends of the string are fixed – boundary conditions have a simple form:

$$u\big|_{x=0} = 0 \qquad u\big|_{x=L} = 0$$

inserting the condition into assumed solution one can obtain

$$\begin{cases} A \cdot 0 + B \cdot 1 = 0 \\ A\sin kL + B\cos kL = 0 \end{cases}$$

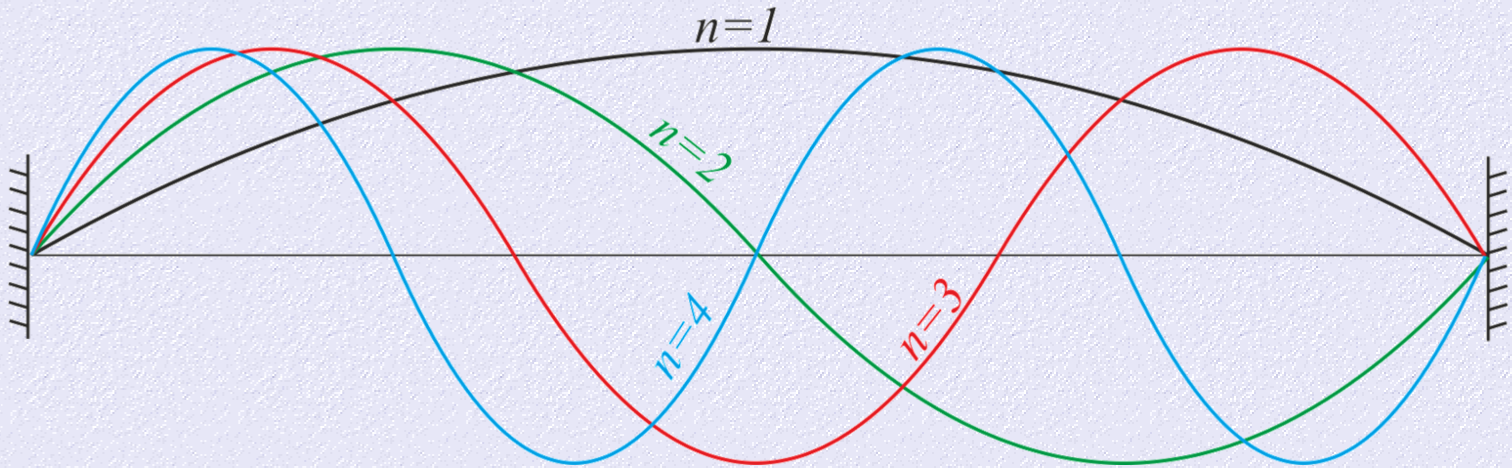
Determinant of the equation system

$$\begin{vmatrix} 0 & 1 \\ \sin kl & \cos kl \end{vmatrix}$$

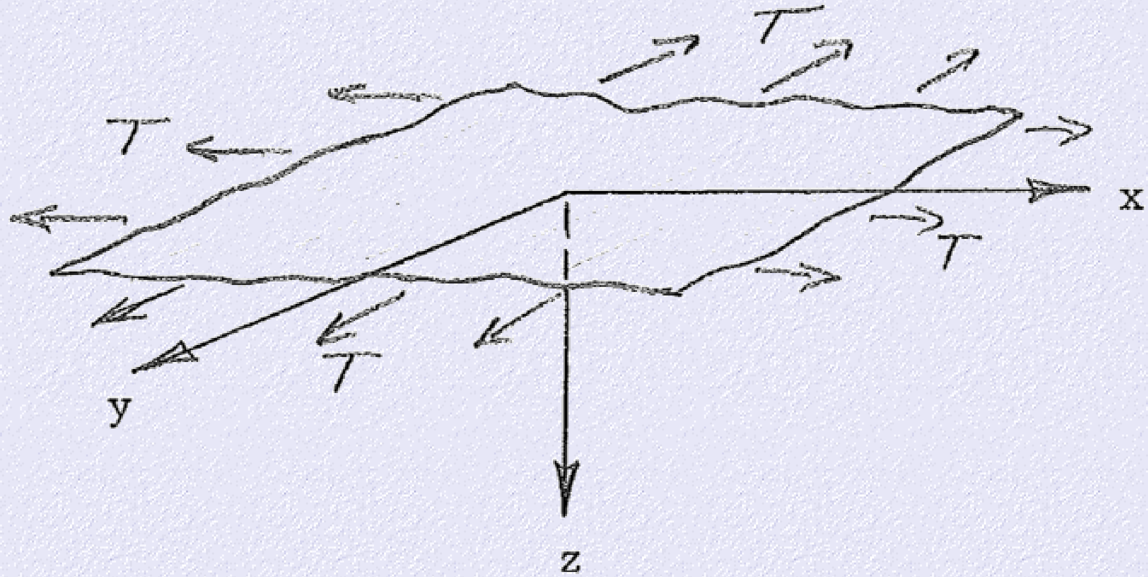
Allows to obtain eigenvalues (possible resonant frequencies of the string)

$$\sin kl = 0 \longrightarrow k_n = \frac{n\pi}{l} \qquad n = 1, 2, 3 \ldots$$

because $$k = \frac{2\pi}{\lambda} \longrightarrow \lambda_n = \frac{2l}{n} \longrightarrow f_n = c\frac{n}{2l} = \sqrt{\frac{T}{\rho}}\frac{n}{2l}$$

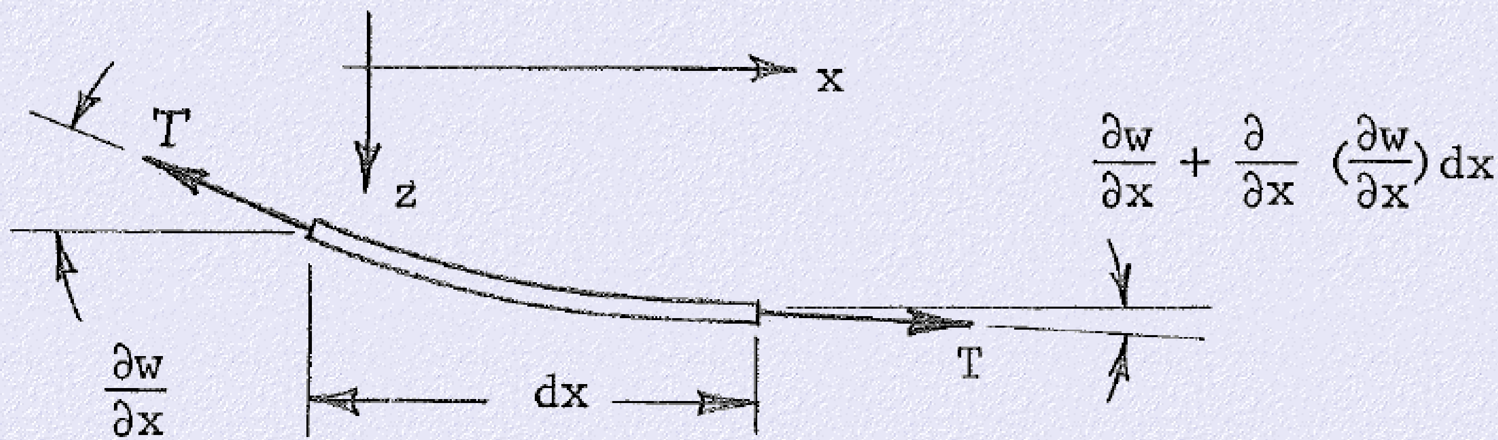


2D case - membranes



The transverse deflection measure $w(x,y,t)$ is in the z-direction.

Isolating a dx by dy element of membrane, and viewing along the y-axis shows



A similar view occurs along the x-axis

Using Newton's Law in the z-direction gives

$$T dy \frac{\partial^2 w}{\partial x^2} dx + T dx \frac{\partial^2 w}{\partial y^2} dy = \rho dx dy \frac{\partial^2 w}{\partial t^2}$$

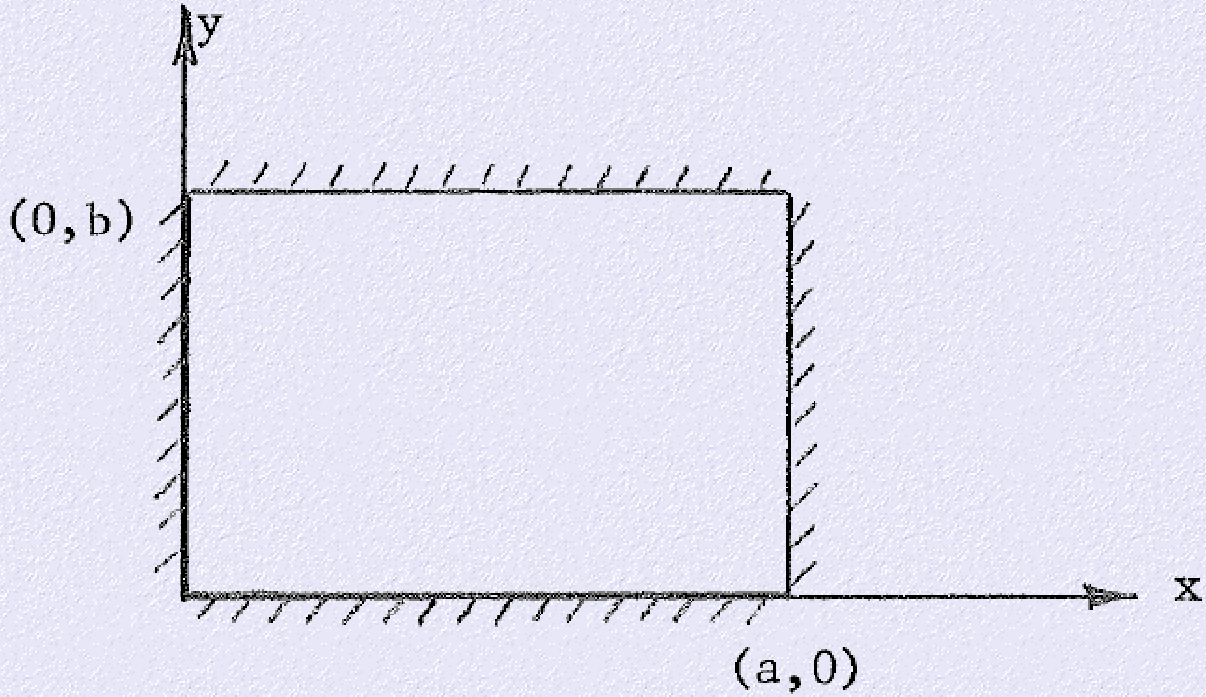
Dividing by dxdy, we can rewrite the above as

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2}$$

where $c = \sqrt{\frac{T}{\rho}}$

T = tension/length

ρ = mass/area



$$w_{xx} + w_{yy} = \frac{1}{c^2} w_{tt}$$

$$w(0,y,t) = w(a,y,t) = 0$$

$$w(x,0,t) = w(x,b,t) = 0$$

$$w(x,y,0) = f(x,y)$$

$$\dot{w}(x,y,0) = 0$$

Using separation of variables

$$w(x,y,t) = X(x)Y(y)T(t)$$

$$X'' + \alpha^2 X = 0$$

$$Y'' + \beta^2 Y = 0$$

$$\ddot{T} + (\alpha^2 + \beta^2)c^2\ T = 0$$

α and β are separation constants

The solutions to equations

$$X = A \sin\alpha x + B \cos\alpha x$$

$$Y = H \sin\beta y + D \cos\beta y$$

$$T = E \sin c\sqrt{\alpha^2 + \beta^2}\ t + F \cos c\sqrt{\alpha^2 + \beta^2}\ t$$

A, B, H, D, E and F being constants

Using the boundary conditions

$$B = D = 0$$

$$\alpha = \frac{m\pi}{a} \quad ; \; m = 1, 2, \; \ldots$$

$$\beta = \frac{n\pi}{b} \quad ; \; n = 1, 2, \; \ldots$$

$$E = 0$$

Consequently applying superposition

$$w(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos\left[\pi c t \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}\,\right] \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

from initial condition t=o

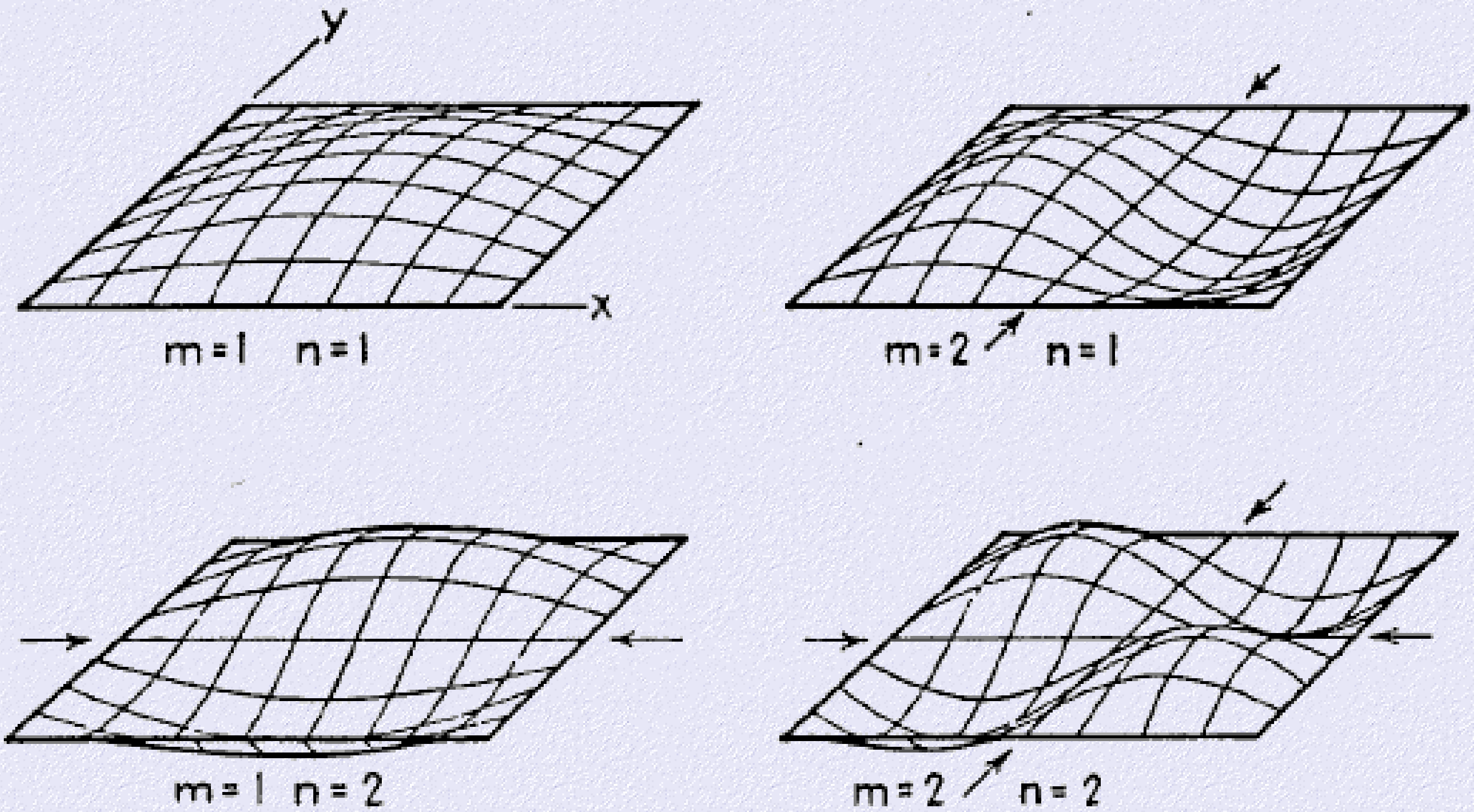
$$f(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

it is double Fourier sine series for f(x,y), so

$$A_{mn} = \frac{4}{ab} \int_0^a \int_0^b f(\xi,\eta) \sin \frac{m\pi\xi}{a} \sin \frac{n\pi\eta}{b} \, d\xi d\eta$$

$$f = \frac{\omega}{2\pi} = \frac{c}{2} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

Note that n=0 or m=0 give the frequency for vibrating string



Zero deflection lines are called nodal lines

An axisymmetric problem – circular membrane with radius r

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} = \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} \quad , \quad w = w(r,t)$$

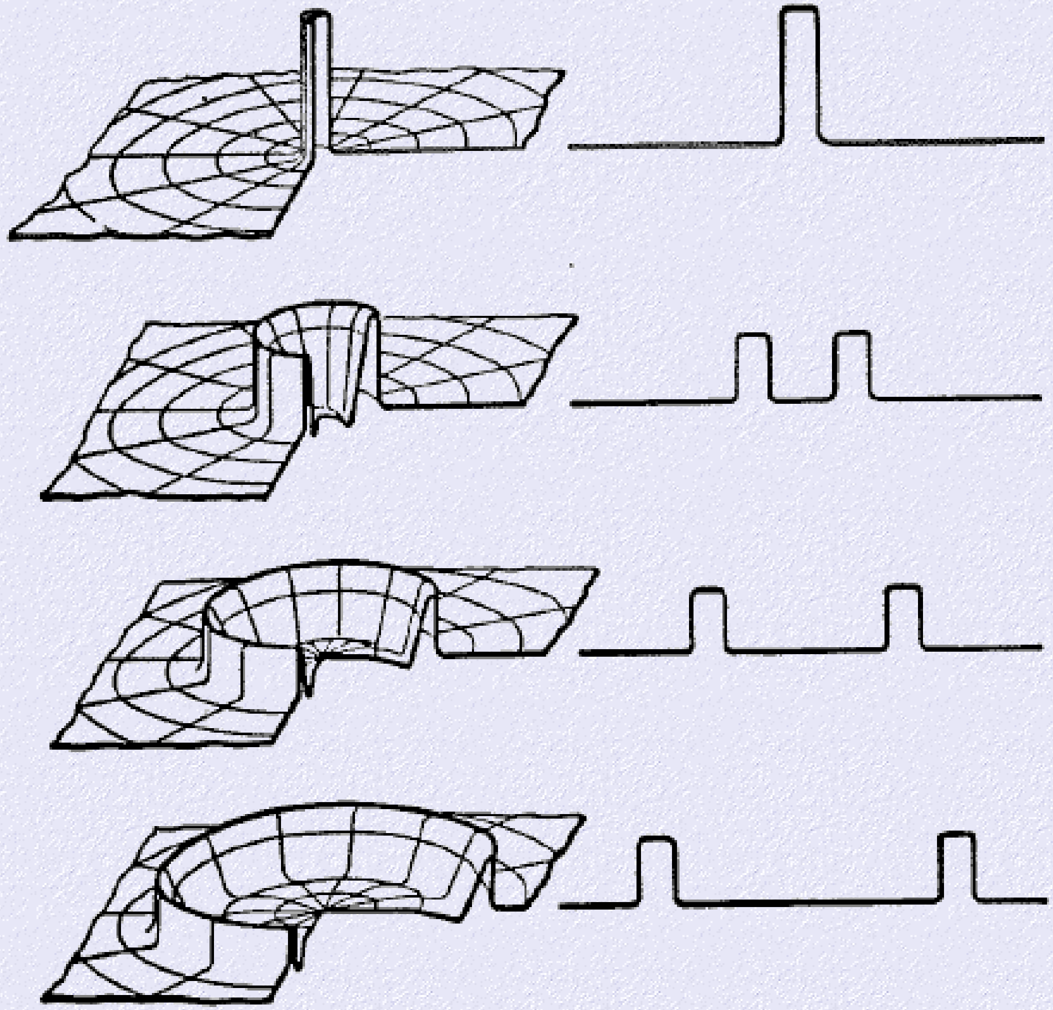
using boundary and initial conditions are simpler

$$w(r,0) = f(r)$$

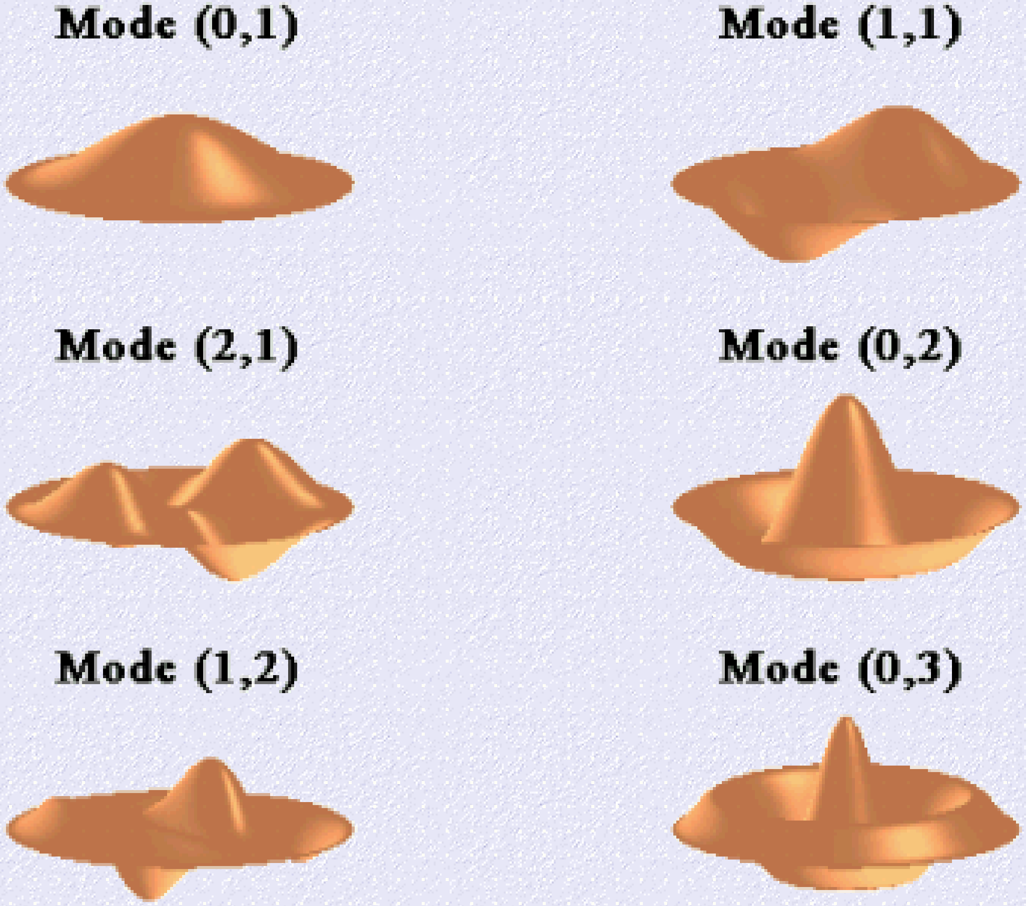
$$w_t(r,0) = 0$$

allow to obtain the solution

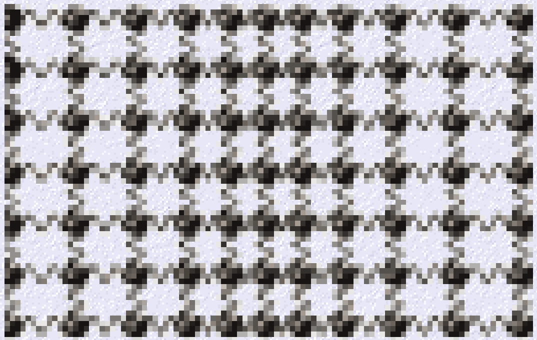
$$w(r,t) = -\frac{1}{2\pi}\frac{ct}{r^{1/2}(c^2t^2 - r^2)^{3/2}} \quad , \quad 0 < r < ct$$



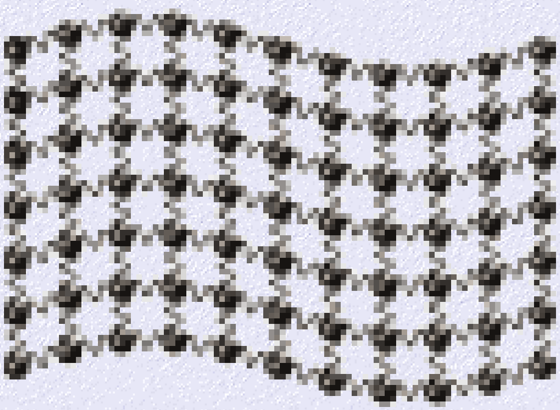
For more complex deflections



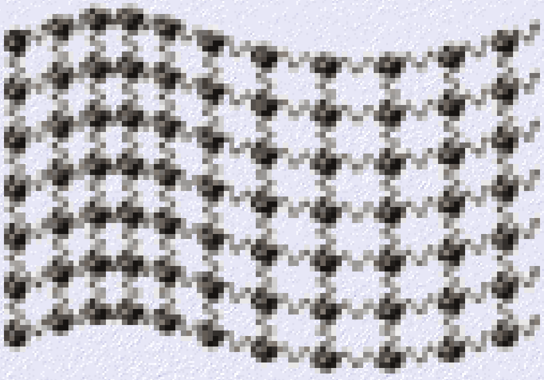
Inside the volume of the solids



longtitudinal waves



transversal waves



both

Velocity is described by Newton formula

$$c = \sqrt{\frac{E}{\rho}}$$

E - Young's modulus fot lingtitudinal waves
(Kirchhoff modulus G for transversal wave)

From definition $$E = \frac{F}{\mathrm{S}} : \frac{\Delta l}{l}$$ so $$\frac{\Delta l}{l} = \frac{1}{E} \cdot \frac{F}{\mathrm{S}}$$

F acting force, S area of force action, *l* length of medium (elestic body), Δ*l* displacement.

Ratio *F/*S is called tension (*T*), and Δ*l*/*l* strain or deformation (*S*) so:

$$S = \frac{1}{E} T$$

It is Hooke low – proper for small displacements and smal viscosities

Let us consider rod having cross section area equal to S and length l

In certain moment t displacement in the point x is equal to u and in the point $x+dx$ *is* $u+du$. The displacements are caused by tensions T and $T+\Delta T$, *respectively*.
The rod displacement toward the distance dx changes to du:

$$\frac{\Delta l}{l} = \frac{d\mathbf{u}}{dx} = \frac{\partial \mathbf{u}}{\partial x} \qquad \text{so} \qquad d\mathbf{u} = \frac{\partial \mathbf{u}}{\partial x} dx$$

From Hooke low:

$$\frac{\partial u}{\partial x} = \frac{1}{E} T$$

The motion of mass element of the rod between *x* and *x*+*dx* is equal to

$$dm = S\rho dx$$

Multiply both sides by acceleration operator one can obtain:

$$dm\frac{\partial^2 u}{\partial t^2} = S\rho dx\frac{\partial^2 u}{\partial t^2}$$

From II Newton dynamic principle left side of the equation is equal to force acting at mass element *dm*.
On the other hand the force actin at surface area makes the tension so:

$$F = \mathrm{S}(T + dT) - \mathrm{S}T = \mathrm{S}\left(T + \frac{\partial T}{\partial x}dx\right) - \mathrm{S}T = \mathrm{S}\frac{\partial T}{\partial x}dx$$

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial T}{\partial x}$$

$$\frac{\partial \boldsymbol{u}}{\partial x} = \frac{1}{E} T \qquad / \cdot \partial x$$

$$\frac{\partial^2 \boldsymbol{u}}{\partial x^2} = \frac{1}{E} \frac{\partial T}{\partial x}$$

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \frac{\partial T}{\partial x}$$

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = E \frac{\partial^2 \boldsymbol{u}}{\partial x^2}$$

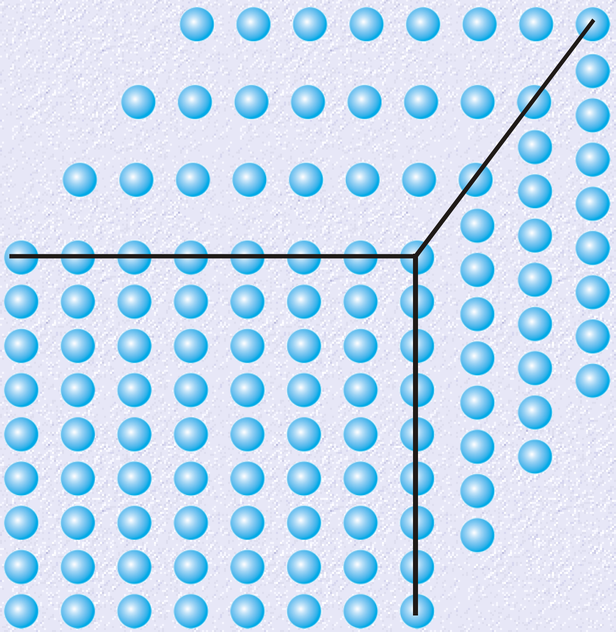
From Newton formula $\quad c = \sqrt{\frac{E}{\rho}}$

$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = c^2 \frac{\partial^2 \boldsymbol{u}}{\partial x^2}$$

It is one dimensional wave equation. It is easy to generalise this equation for 3 D case (or even n D cases)

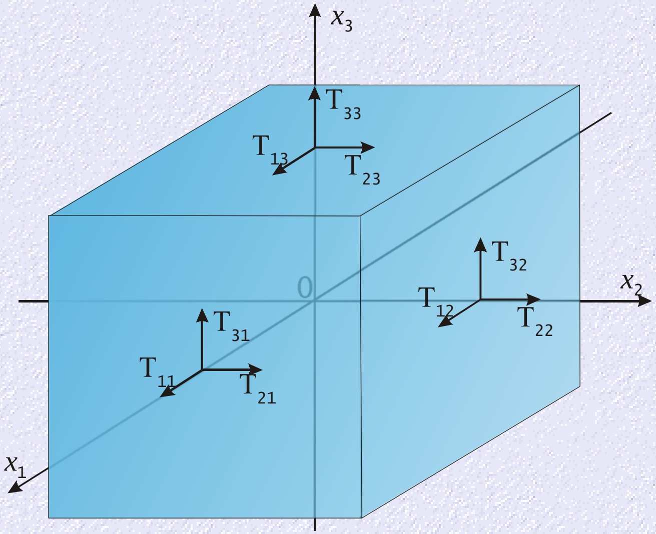
$$\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = v^2 \left(\frac{\partial^2 \boldsymbol{u}}{\partial x^2} + \frac{\partial^2 \boldsymbol{u}}{\partial y^2} + \frac{\partial^2 \boldsymbol{u}}{\partial z^2} \right) = v^2 \nabla^2 \boldsymbol{u}$$

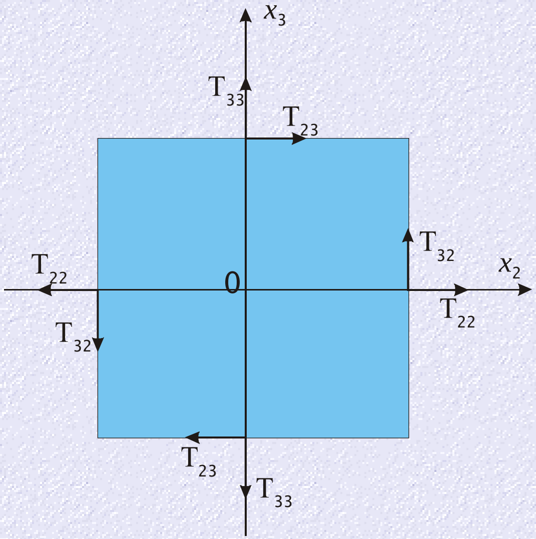
The Hooke low $S = \frac{1}{E} T$ operates properly for one fixed direction



For anizotropy the wave velocities depend on direction of the propagation

About 1910 r. Woldemar Voigt had introduced tensor algebra to take into consideration all directions

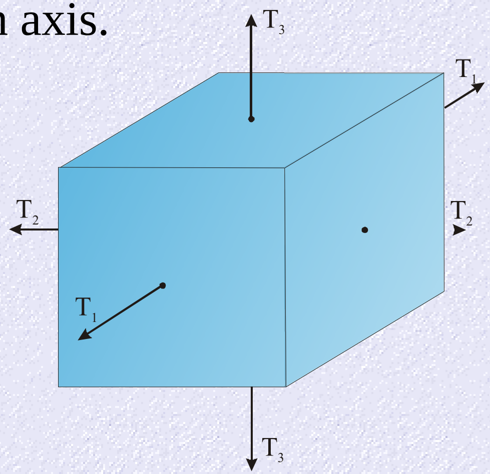




The unit cube is small enough to make the internal tensions homogeneous.
The tensions T_{ii} are compressive and T_{ij} shear.

The symmetry of the tensions allows to bring the unit cube to main axis.
It is the situation when shear stresses vanish.

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \to \begin{bmatrix} T_1 & 0 & 0 \\ 0 & T_2 & 0 \\ 0 & 0 & T_3 \end{bmatrix}$$



It is possible if the coordinate system is chosen in the way when the directions of of unit cube edges and axis are the same.

In general case for anizotropic solids Hooke low has more general form:

$$T_{ij} = \sum_k \sum_l C_{ijkl} S_{kl}$$

Summation sign may be ommited when summation is over a set of indexed terms in a formula, thus achieving notational brevity (Einstein summation convention).

$$T_{ij} = C_{ijkl} S_{kl}$$

C_{ijkl} stifness tensor (or elestic constants) that characterises elestic properties of the solid In anizotrpic case.

The strain tensor S_{kl} is symmetric:

$$S_{kl} = S_{lk}$$

The fact reduces number of independent components from 9 to 6

Symmetry of S_{kl} imply symmetry of C_{ijkl}

$$C_{ijkl} = C_{jikl} \quad C_{ijkl} = C_{klij} \quad C_{ijkl} = C_{ijlk}$$

It reduces number of components from 81 to 21.
For cubic crystals (perovskites) the number od independent components reduces to 3.

In the special izotropic case the elastic constant tensor reduces to two components so called Lamé constants λ i μ

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right)$$

$$\delta_{ij} = \begin{cases} 1 \text{ dla } i = j \\ 0 \text{ dla } i \neq j \end{cases}$$

Hooke low is independent of time but it is easy to dynamise it using II Newton dynamic principle

$$\boldsymbol{F} = m\boldsymbol{a}$$

From definition $F_i = \frac{\partial T_{ij}}{\partial x_j} \longrightarrow \rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j}$ $\qquad m = \rho V$

$$T_{ij} = C_{ijkl} S_{kl} \qquad S_{ij}(x_1, x_2, x_3) = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \longrightarrow T_{ij} = C_{ijkl} \frac{\partial u_j}{\partial x_k}$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial T_{ij}}{\partial x_j} \qquad T_{ij} = C_{ijkl} \frac{\partial u_j}{\partial x_k} \longrightarrow \qquad \rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k}$$

It is the wave equation for function Ψ, that has in general form of plane wave. Every displacement inside solid volume is a source of acoustic wave

3D Laplace operator

$$\frac{\partial^2 \Psi}{\partial t^2} = v^2 \nabla^2 \Psi \qquad \nabla^2 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3}$$

General solution

$$\boldsymbol{u} = A e^{\left[i\boldsymbol{k}\left(l_i x_i - vt \right) \right]}$$

Uniform planar waves of this kind are called bulk acoustic waves (BAW).

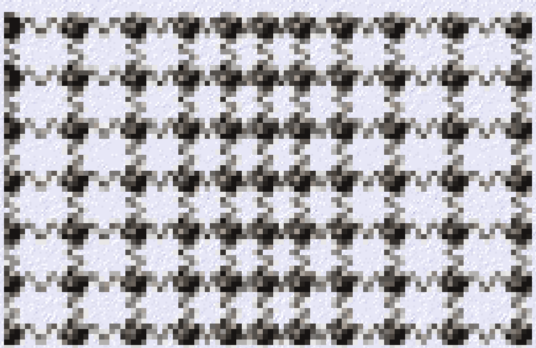
Inserting the solution into wave equation gives 3 waves propagating in 3 directions: one longtitutinal and two transversal (with the same velocities)

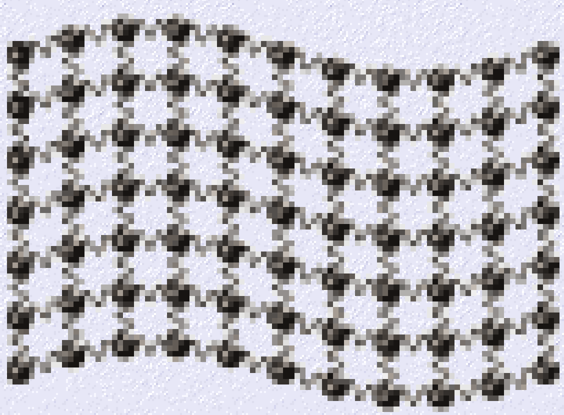
$$v_l = \sqrt{\frac{C_{1111}}{\rho}}$$

$$v_{t1} = v_{t2} = \sqrt{\frac{C_{2323}}{\rho}}$$

Newton formula!

For most solids $$v_t \approx 0,63 v_l$$





For izotropy $$C_{1111} = \lambda + 2\mu,\ C_{1122} = \lambda,\ C_{2323} = \mu$$

or using Voigt notation $11 \to 1,\ \ 22 \to 2,\ \ 33 \to 3,\ \ 23 \to 4,\ \ 13 \to 5,\ \ 12 \to 6$

$$C_{11} = \lambda + 2\mu,\ C_{12} = \lambda,\ C_{44} = \mu$$

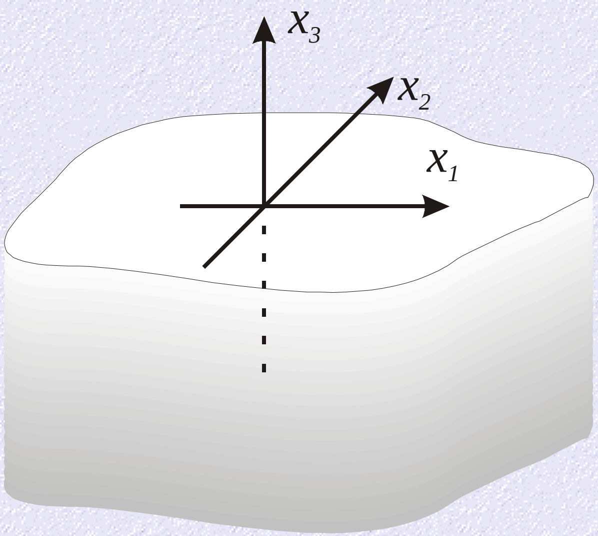
$$v_l = \sqrt{\frac{C_{1111}}{\rho_0}} \qquad v_t = \sqrt{\frac{C_{44}}{\rho_0}}$$

$$v_l = \sqrt{\frac{\lambda + 2\mu}{\rho_0}} \qquad v_t = \sqrt{\frac{\mu}{\rho_0}}$$

There are oure longtidudinal and transversal waves.
in general inside anizotropic solids tipical displacements directions of particles are not
Parallel or transversal to the propagation directions (are not pure).

Each limit of space (each border) can guide new kind of waves

Cutting the infinitive elastic space by plane one can obrtain two elastic half-spaces



Let us look for planar waves (displacement u_2=0) propagating along x_1 and vanishing into The depth of the half-space

$$v_t^2\nabla^2 u_1 + (v_l^2 - v_t^2)u_{1,11}u_{3,31} - u_{1,tt} = 0$$

$$v_t^2\nabla^2 u_3 + (v_l^2 - v_t^2)u_{1,13}u_{3,33} - u_{3,tt} = 0$$

due to u_2=0 laplacian has 2D form $\nabla^2 = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_3}$

Let us represent the displacement vector using two potencial functions

$$\boldsymbol{u} = grad\,\varphi + rot\,\Psi$$

scalar potential
nonrotational part

vector potential
rotational part

For planar vawes we obtain:

$$u_1 = \varphi_{,1} + \psi_{,3}$$

$$u_3 = \varphi_{,3} + \psi_{,1}$$

Inserting displacements into wave equations we obtain

$$v_l^2 \nabla^2 \varphi - \varphi_{,tt} = 0$$

$$v_t^2 \nabla^2 \psi - \psi_{,tt} = 0$$

Boundary conditions

$$T_{33} = 0$$

$$T_{31} = 0$$

From above

$$T_{33} = (\lambda + 2\mu)\varphi_{,33} + \lambda\varphi_{,11} - 2\mu\psi_{,13} = 0$$

$$T_{31} = \mu\left(2\varphi_{,13} - \psi_{,11} + \psi_{,33}\right) = 0$$

Looking for planar wave along x_1 that vanishes towards x_3

$$\varphi = Ae^{-\alpha x_3 + i(kx_1 - \omega t)} \qquad \psi = Be^{-\beta x_3 + i(kx_1 - \omega t)}$$

one can yeld

$$A\left[(\lambda + 2\mu)\alpha^2 - \lambda k^2\right] + 2iB\mu k\beta = 0$$

$$-2iAk\alpha + B\left(\beta^2 + k^2\right) = 0$$

Vanishing coefficients can be express using wave vector

$$\alpha^2 = k^2 - \frac{\omega^2}{v_l^2}, \quad \beta^2 = k^2 - \frac{\omega^2}{v_t^2}$$

so

$$A\left[(\lambda + 2\mu)\left(k^2 - \frac{\omega^2}{v_l^2}\right) - \lambda k^2\right] + 2iB\mu k\sqrt{k^2 - \frac{\omega^2}{v_t^2}} = 0$$

$$-2iAk\sqrt{k^2 - \frac{\omega^2}{v_l^2}} + B\left(2k^2 - \frac{\omega^2}{v_t^2}\right) = 0$$

Demanding of vanish of the determinant we obtain characteristic equaion:

$$\left(2 - \frac{\omega^2}{k^2 v_t^2}\right)^2 - 4\sqrt{1 - \frac{\omega^2}{k^2 v_l^2}}\sqrt{1 - \frac{\omega^2}{k^2 v_t^2}} = 0$$

or using $v = \frac{\omega}{k}$

$$\left(2 - \frac{v^2}{v_t^2}\right)^2 - 4\sqrt{1 - \frac{v^2}{v_l^2}}\sqrt{1 - \frac{v^2}{v_t^2}} = 0$$

The equatiin has nontrivial solutions for $0 < v < v_t$
The new wave velocity

$$0,874v_t < v < 0,995v_t$$

for compressible solids   for uncompressible solids

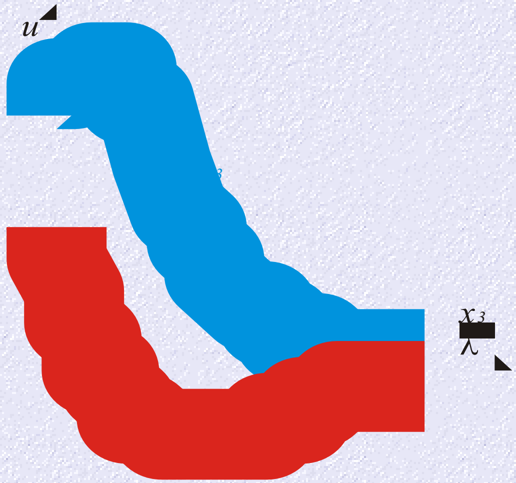
Close the surface can propagate new kind of wave (neither transversal nor longtitudinal).

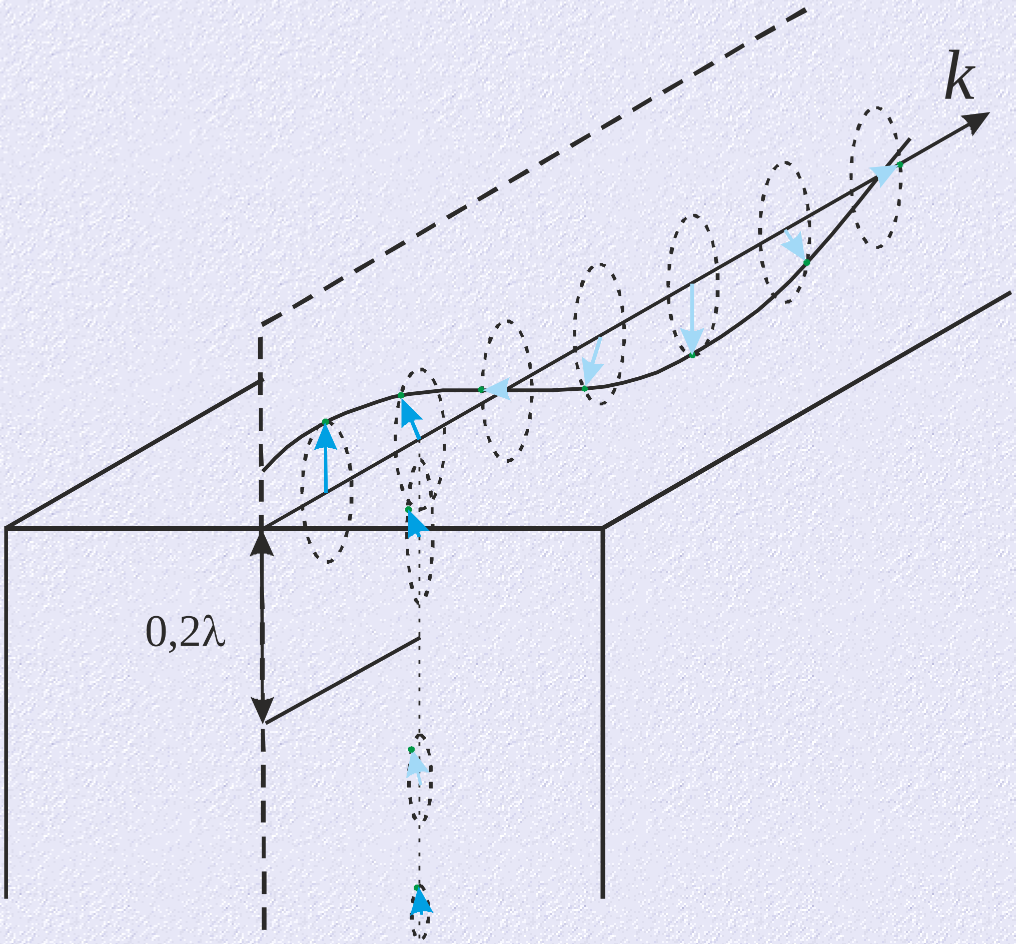
called **Rayleigh wave**.

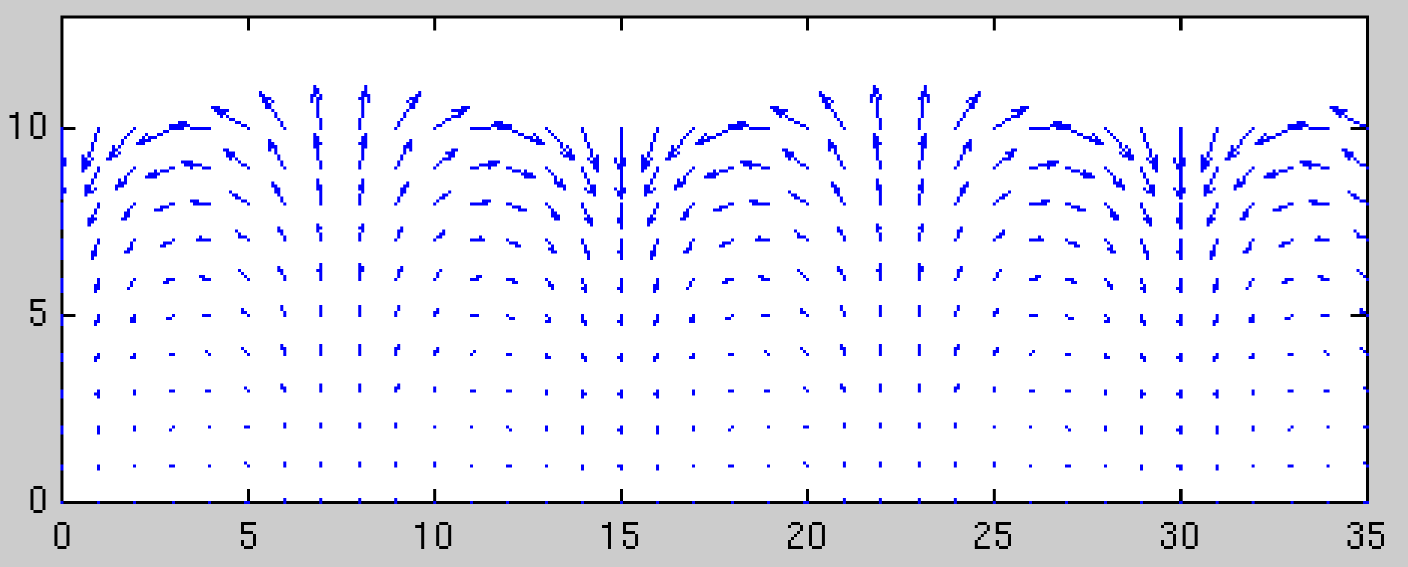
The displacements

$$u_1 = Ae^{-x_3\sqrt{k^2-k_l^2}} \cos\left(kx - vt\right)$$

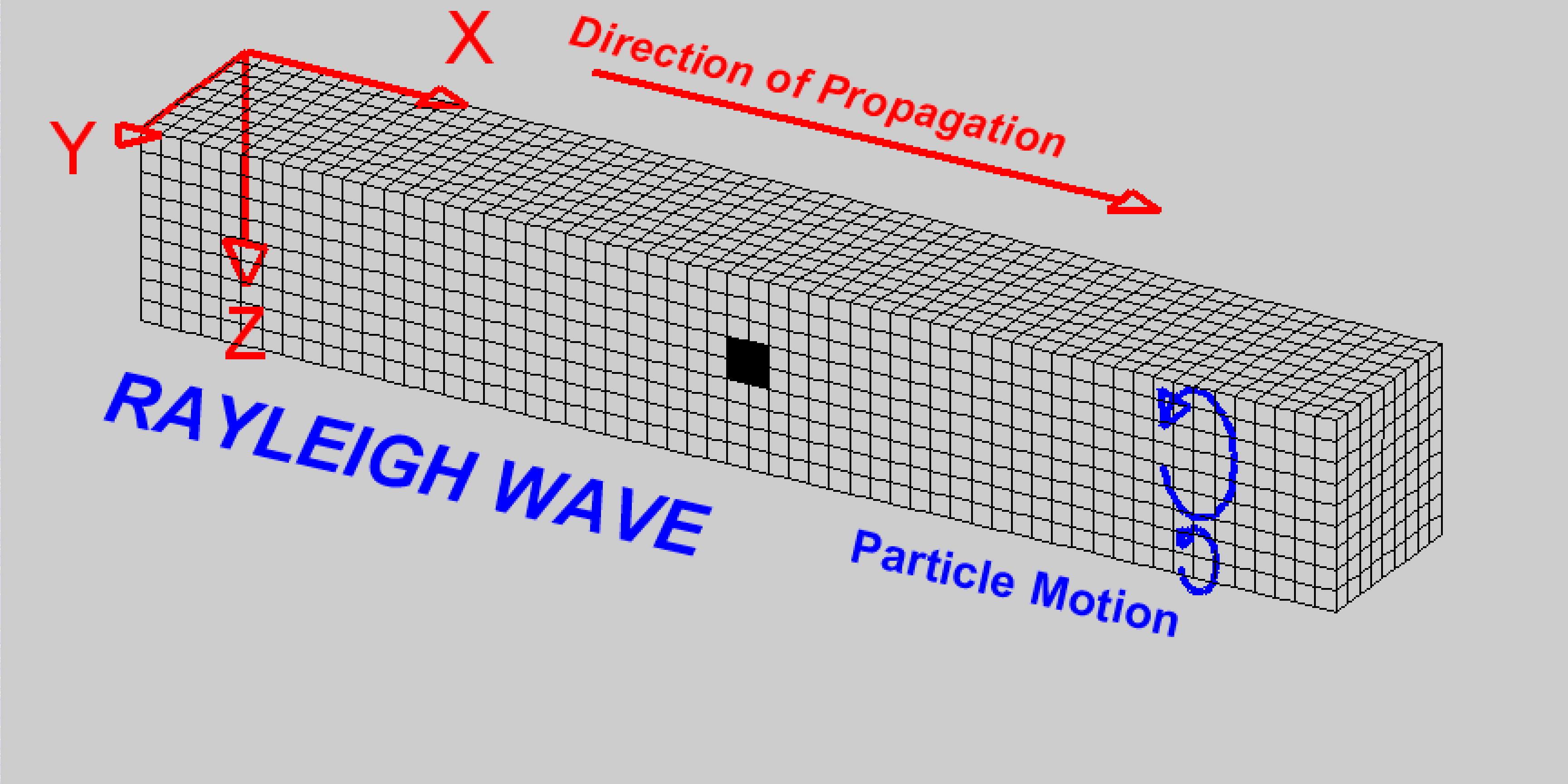
$$u_3 = Be^{-x_3\sqrt{k^2-k_t^2}} \sin\left(kx - vt\right)$$


u
x₃
λ


k
0,2λ



(Rayleigh wave simulation in a halfspace. Dunn, 2001.)

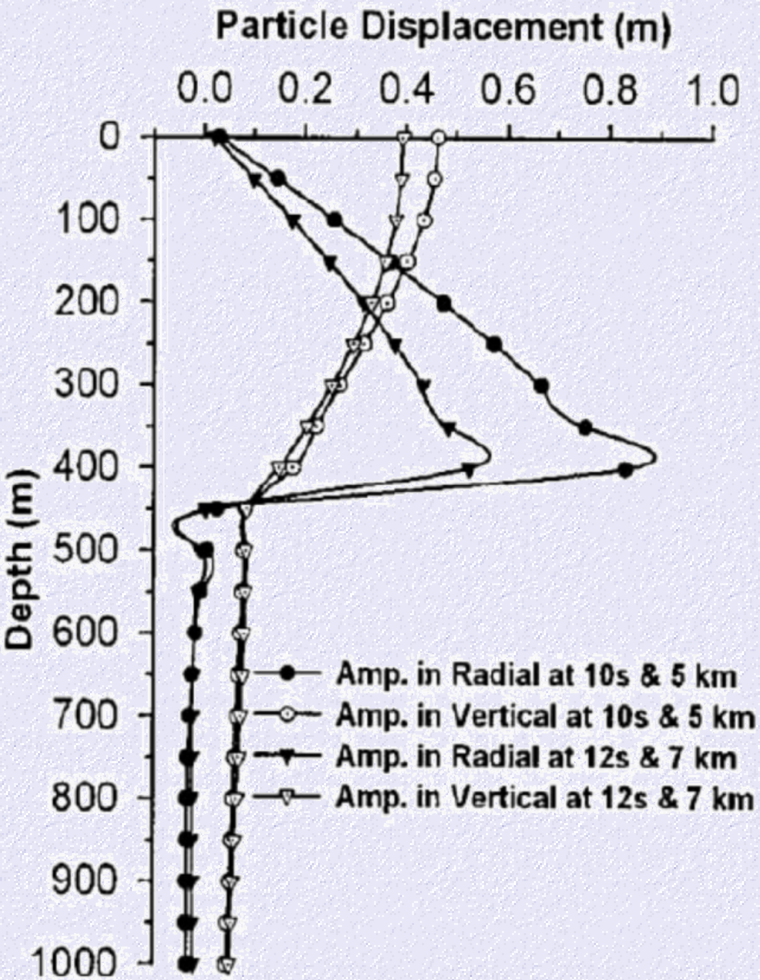

X
Y
Z
Direction of Propagation
RAYLEIGH WAVE
Particle Motion

Rayleigh wave most important properties

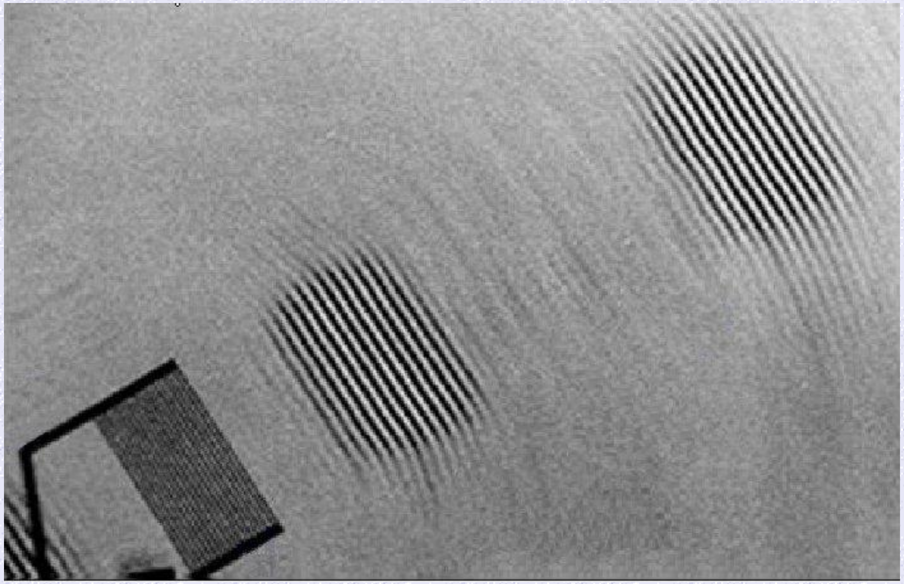
- amplitude vanishes into the depth of half-space on a distance about λ
- amplitude is very small in order even $\sim\lambda^{-5}$
- it has only two components (transversal component = 0)
- eliptic polarisation
- material point at the surface of half-space draw eliptic trajectories in opposite to the wave propagation direction
- it is planar wave – low attenuation
- it is near-surface combination of transversal and longtitudinal modes (Lissajous curve)
- the velocity is lower than bulk longtitudinal and transversal wave

$$v < v_t < v_l$$

$$0,874 v_t < v < 0,995 v_t$$

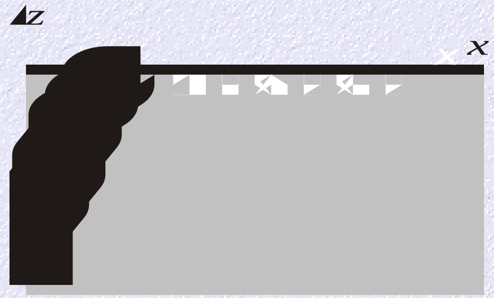


Particle Displacement (m)
0.0
0.2
0.4
0.6
0.8
1.0
0
100
200
300
400
500
600
700
800
900
1000
Depth (m)
Amp. in Radial at 10s & 5 km
Amp. in Vertical at 10s & 5 km
Amp. in Radial at 12s & 7 km
Amp. in Vertical at 12s & 7 km

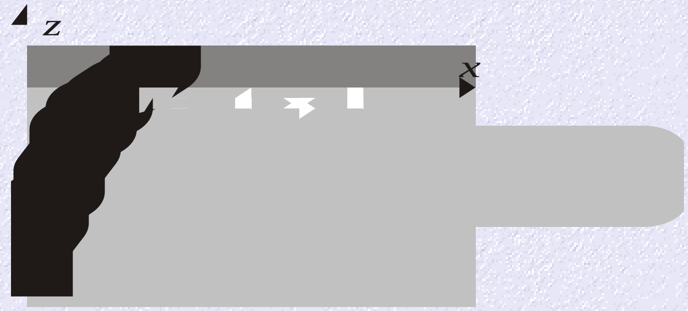


In analog way other possible waves were found

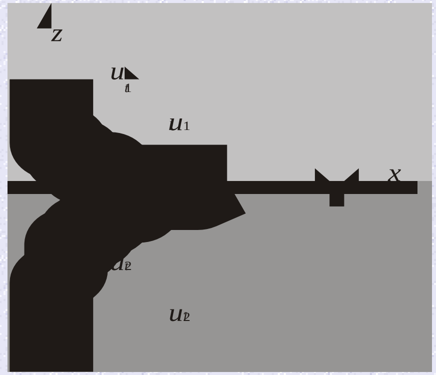
Surface transversal waves



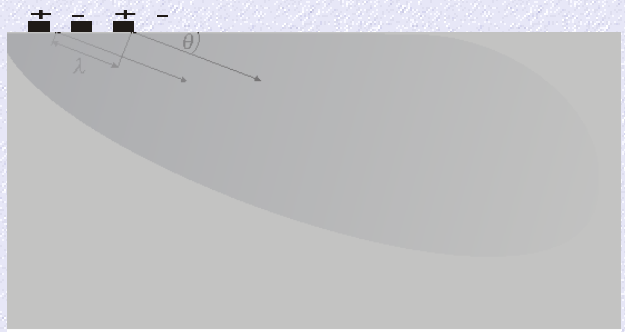
Love waves



Stonley waves

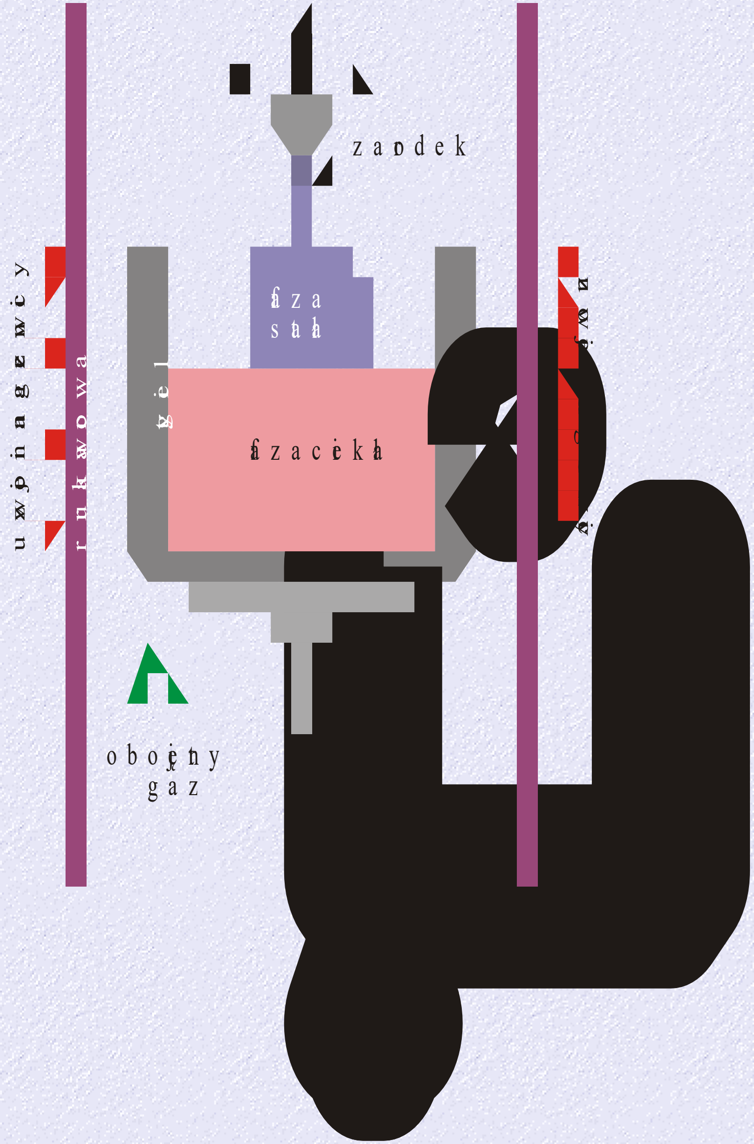


Surface skimming bulk waves (SSBW)

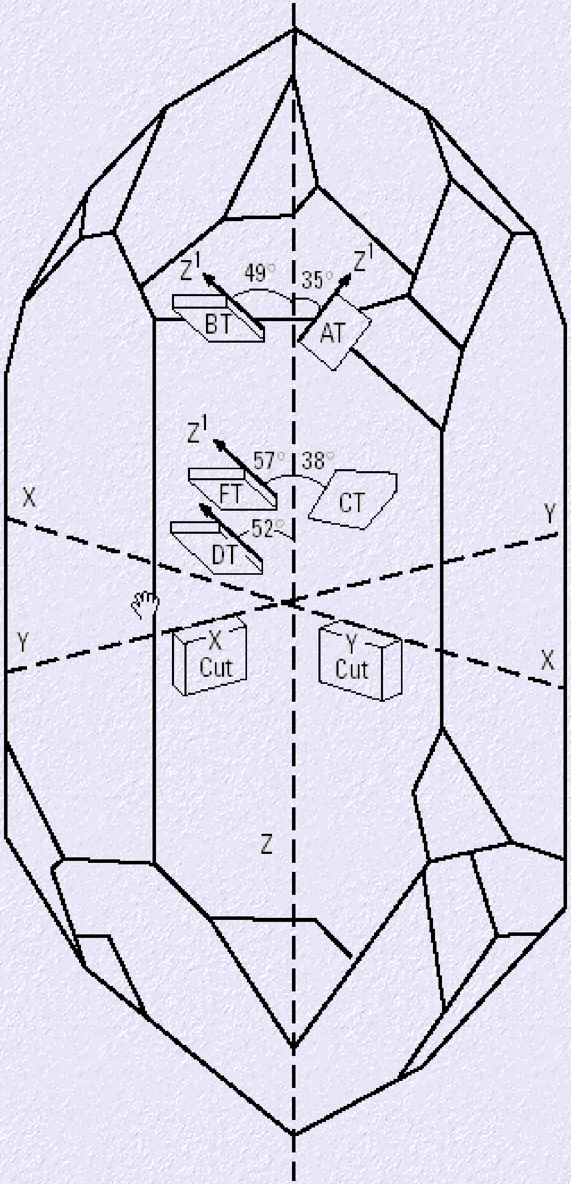


Najsłynniejsze polskie nazwisko
w świecie techniki

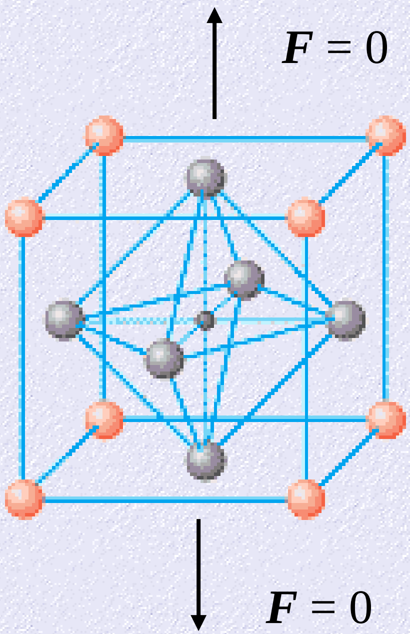
Prof. Jan Czochralski
1885-1953

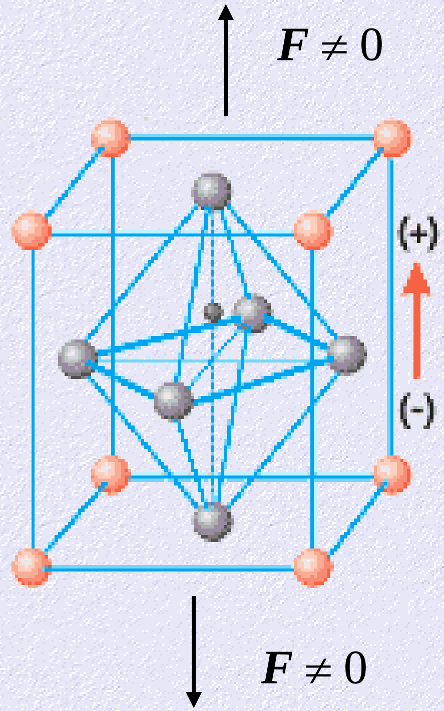


„Wyciąganie” monokryształu z fazy ciekłej metodą Czochralskiego.


Z1
49°
35°
Z1
BT
AT
Z1
57°
38°
FT
CT
52°
DT
X
Y
Y
X
X
Cut
Y
Cut
Z

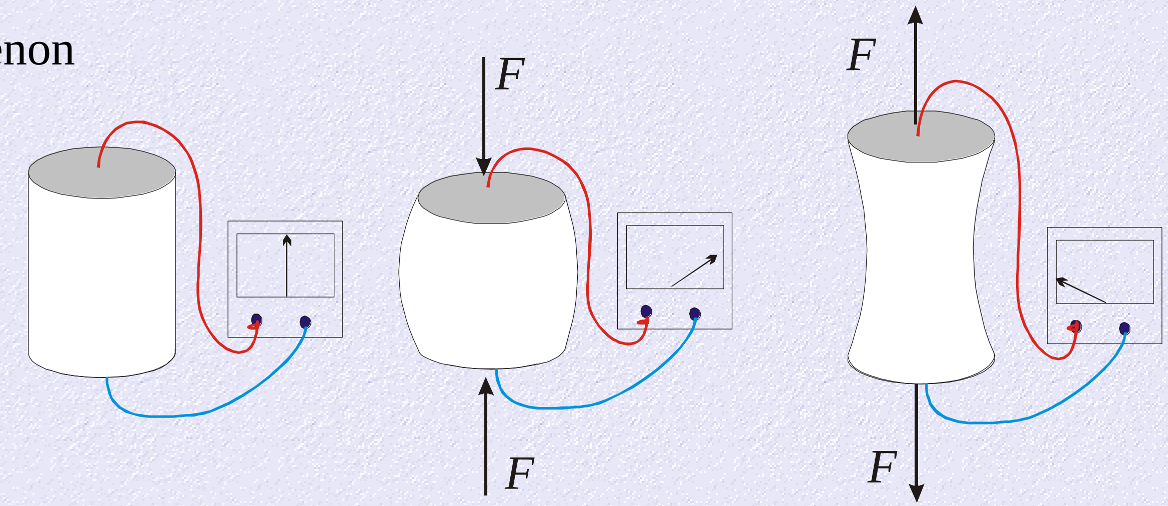
PIEZOELECTRICITY



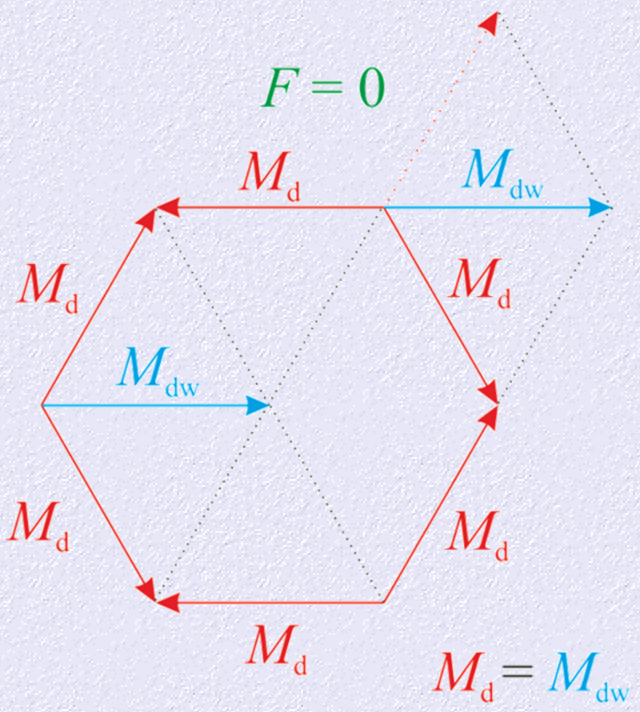


SiO_2 $LiNbO_3$ $LiTaO_3$ $BaTiO_3$ $SrTiO_3$ $Pb(ZrTi)O_3$ $KNbO_3$ $KNaC_4H_4O_6 \cdot 4H_2O$

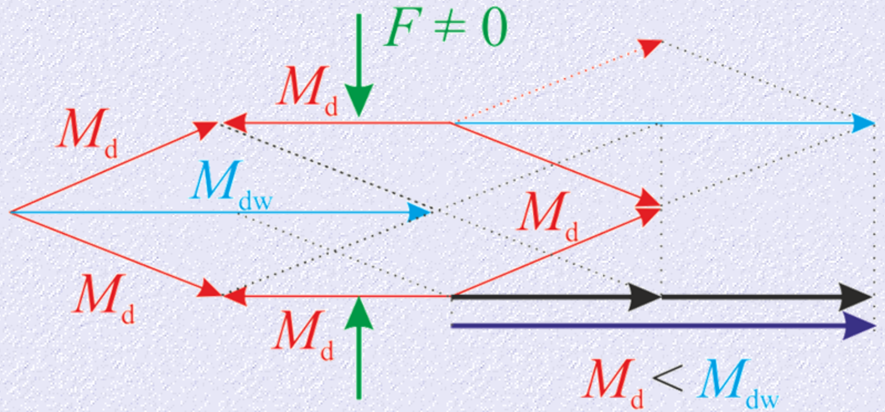
Direct and undirect phenomenon



Geometrical explanation



Full compensation of dipole moments



Dipole moments are uncompensated

Hook low for piezoelectric materials has to be modified

$$T_{ij} = C^{E}_{ijkl} S_{kl} - e_{ijk} E_k$$

e_{ijk} tensor of piezoelectric coefficients

Electric induction has the form

$$D_i = \varepsilon^{S}_{ij} E_j + e_{ijk} S_{jk}$$

ε^{S}_{ij} permittivity tensor at constant strain

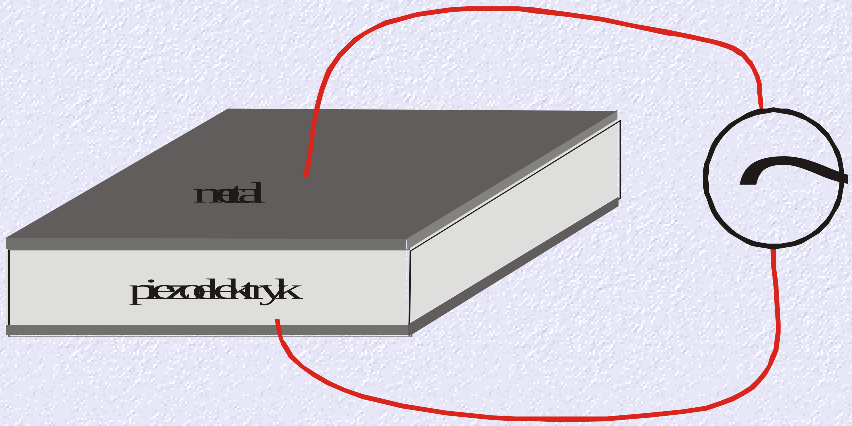
or

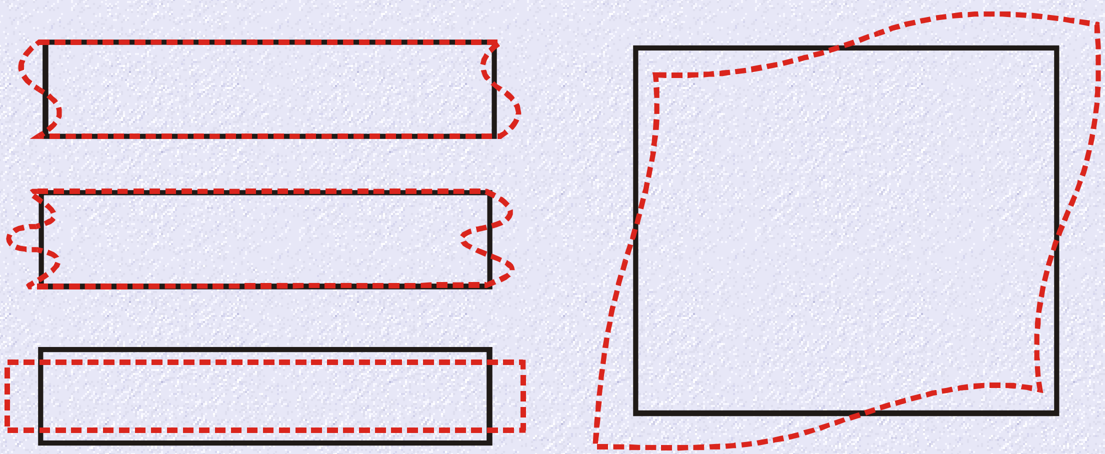
$$D_i = \sum_i \varepsilon^{T}_{ij} E_j + \sum_{j,k} d_{ijk} T_{jk}$$

ε^{T}_{ij} permittivity tensor at constant tension

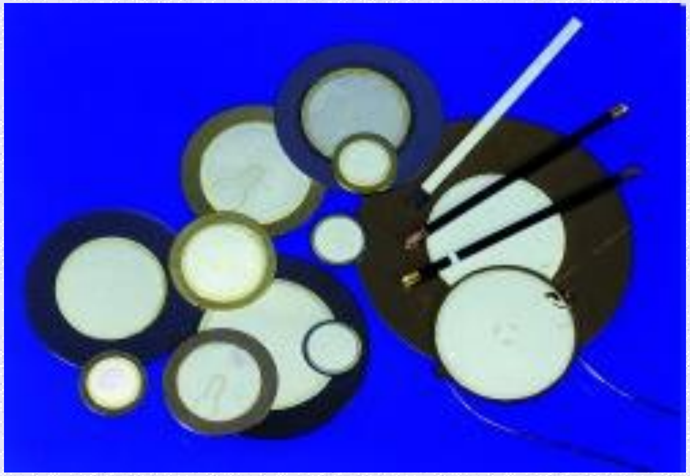
d_{ijk} tensor of direct piezoelectric effect coefficients

Piezoelectric materials allow to excite and detect both bulk and surface waves just using alternating voltage

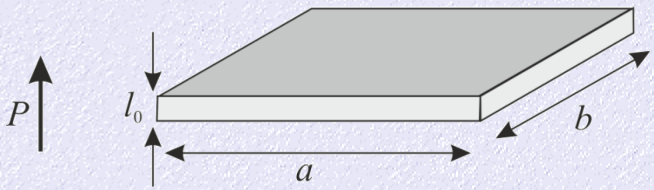




There are gread deal of different shapes

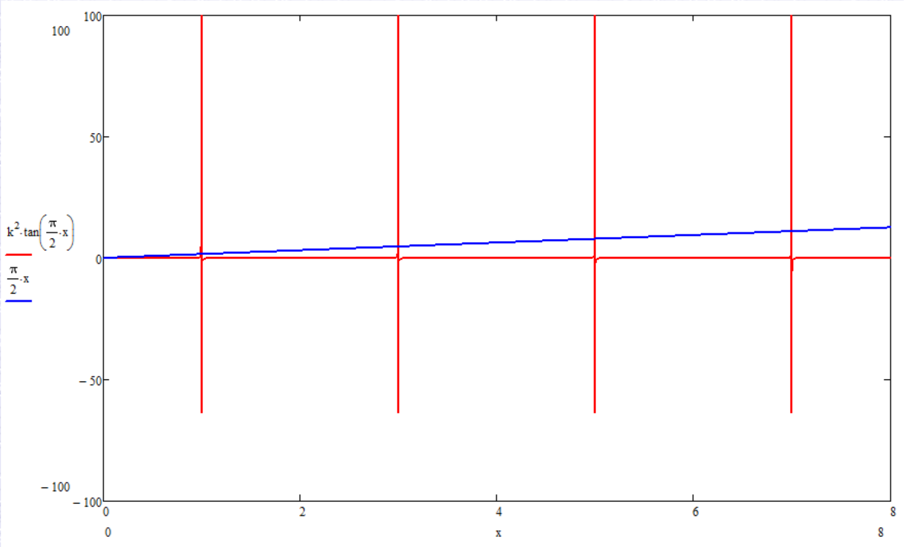


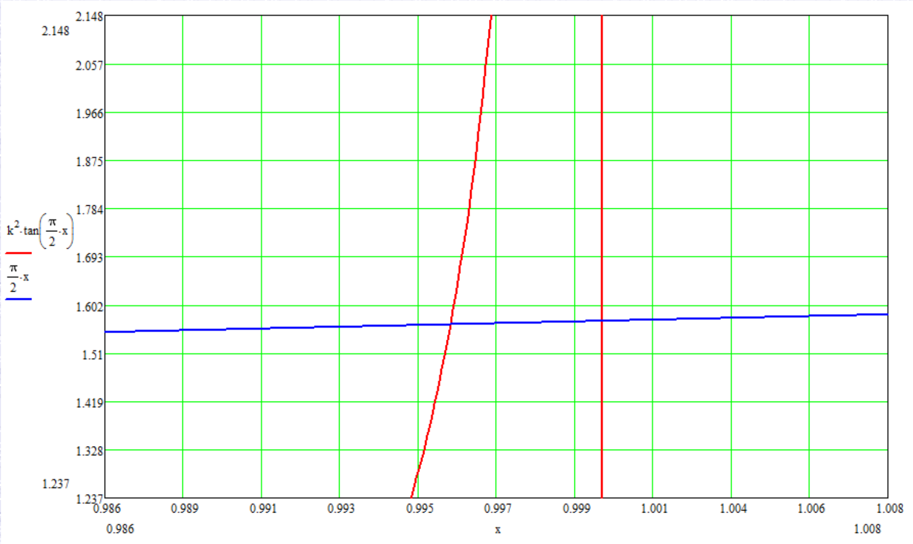
Basic „rigid” ways of vibrations of thin piezoelectric plates (Kazis)



thickness $$f_0 = \frac{1}{l_0}\sqrt{\frac{C_{33}^D}{\rho}}\,x_n = \frac{v\,x_n}{l_0} \qquad n = 1,\,3,\,5\ldots$$

x_n – subsequent roots of the equation $$k_t^2 tg\left(\frac{\pi}{2}x\right) = \frac{\pi}{2}x \qquad k_t = \frac{\varepsilon_{33}}{\sqrt{\varepsilon_{33}^S C_{33}^D}}$$

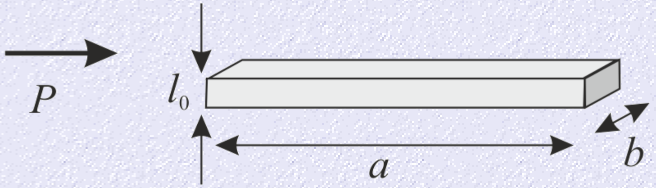




$$x_1 \approx 0{,}996$$

Neded constants are already measured and collected in respective tables

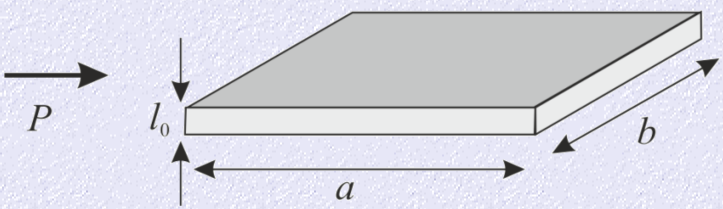
$[c]=$	$$\begin{bmatrix} 86.74 & -8.25 & 27.15 & -3.66 & 0 & 0 \\ -8.25 & 129.77 & -7.42 & 5.7 & 0 & 0 \\ 27.15 & -7.42 & 102.83 & 9.92 & 0 & 0 \\ -3.66 & 5.7 & 9.92 & 38.61 & 0 & 0 \\ 0 & 0 & 0 & 0 & 68.81 & 2.53 \\ 0 & 0 & 0 & 0 & 2.53 & 29.01 \end{bmatrix}$$ $\times 10^{9}\ \mathrm{N/m^2}$
$[e]=$	$$\begin{bmatrix} 17.1 & -15.2 & -1.87 & 6.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10.8 & -9.5 \\ 0 & 0 & 0 & 0 & -7.61 & 6.7 \end{bmatrix}$$ $\times 10^{-2}\ \mathrm{C/m^2}$
$[\varepsilon]=$	$$\begin{bmatrix} 39.21 & 0 & 0 \\ 0 & 39.82 & 0.86 \\ 0 & 0.86 & 40.42 \end{bmatrix} \times 10^{-12}\ \mathrm{C/Vm}$$



longtitudinal vibrations of rod

$$f_0 = \frac{1}{2l_0\sqrt{S_{33}^D\rho}}x_n = \frac{v\,x_n}{l_0} \qquad n = 1,\ 3,\ 5...$$

$$x_n \Rightarrow \quad k_{33}^2 tg\left(\frac{\pi}{2}x\right) = \frac{\pi}{2}x \qquad k_{33} = \frac{d_{33}}{\sqrt{\varepsilon_{33}^T S_{33}^E}}$$

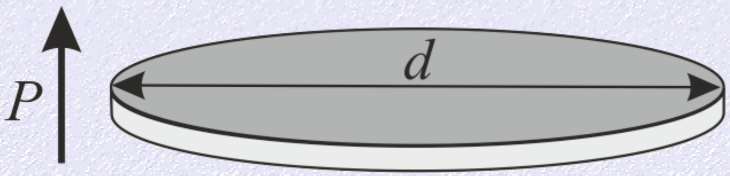


Shear vibrations

$$f_0 = \frac{1}{2l_0}\sqrt{\frac{C_{44}^D}{\rho}}x_n = \frac{v\,x_n}{l_0} \qquad n = 1,\ 3,\ 5...$$

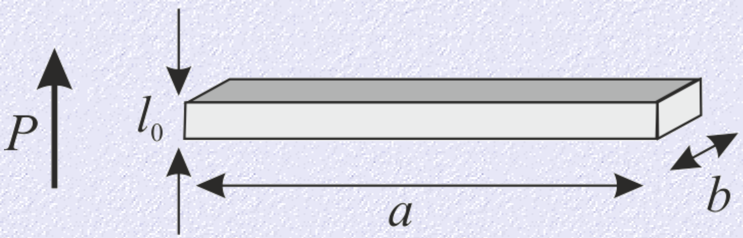
$$x_n \Rightarrow \quad k_{15}^2 tg\left(\frac{\pi}{2}x\right) = \frac{\pi}{2}x \qquad k_{15} = \frac{e_{15}}{\sqrt{\varepsilon_{11}^S C_{44}^D}}$$

Basic „free” ways of vibrations (Kazis)



radial

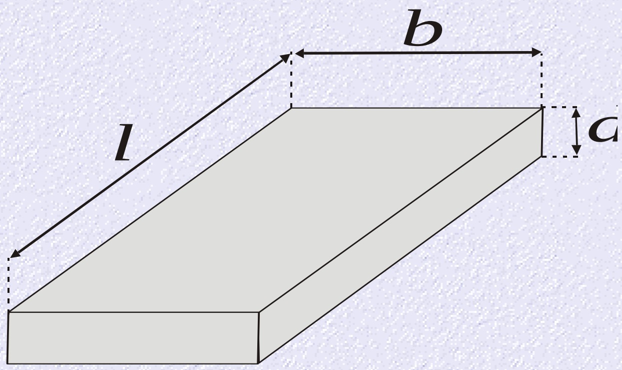
$$f_0 = \frac{\xi}{2\pi d\sqrt{S_{11}^E \rho\left[1-\left(T^E\right)^2\right]}} = \frac{\xi v}{2\pi d}$$

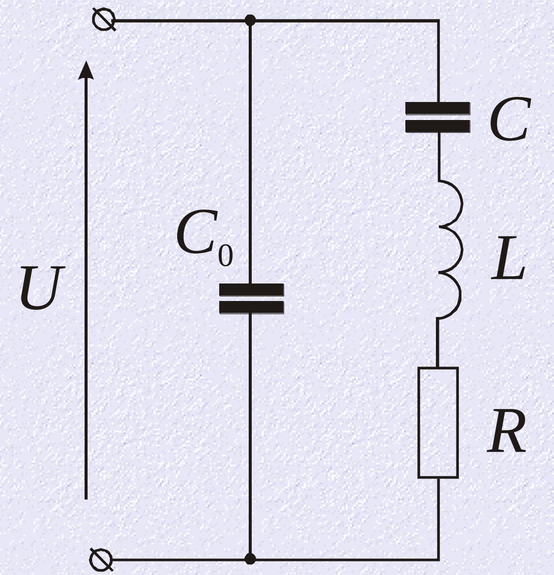


longtitudinal

$$f_0 = \frac{n}{2a\sqrt{S_{11}^E \rho}} = \frac{nv}{2a} \qquad n = 1, 3, 5...$$

Equivalent circuit





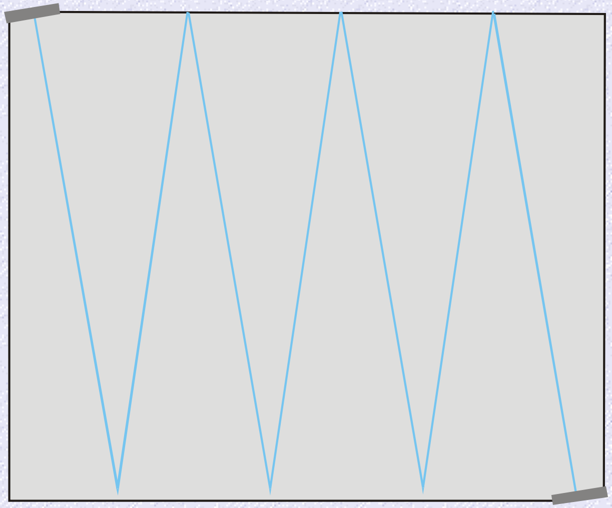
$$\alpha \sim \frac{bl}{dd_{11}}$$

$$L = \frac{m}{\alpha^2} \qquad R = \frac{r}{\alpha^2} \qquad C = \frac{\alpha^2}{k}$$

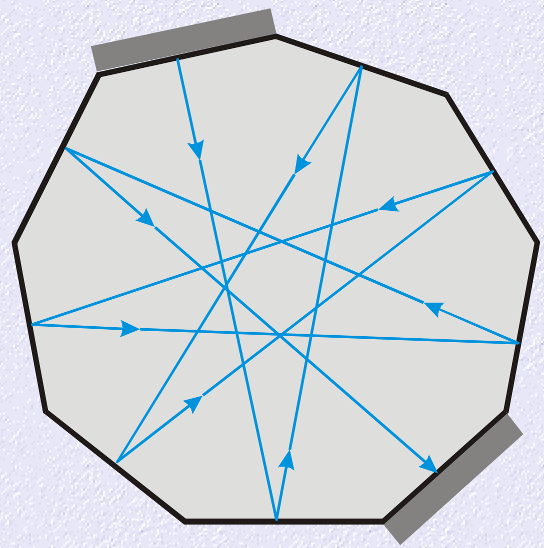
m, *r*, *k*, depend on piezoelectric material and details of the construction

Basic BAW devices

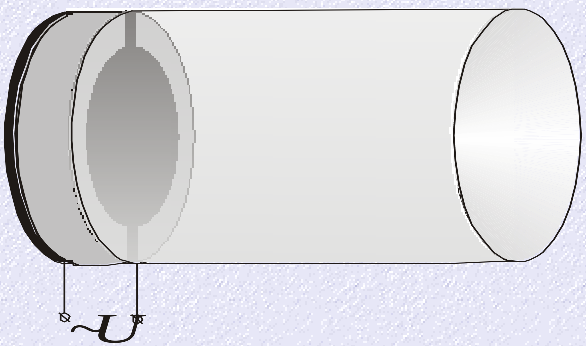
Delay lines



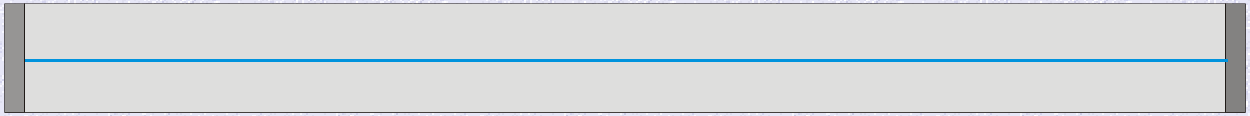
64 μs



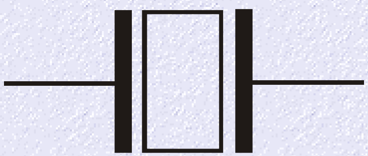
120 μs

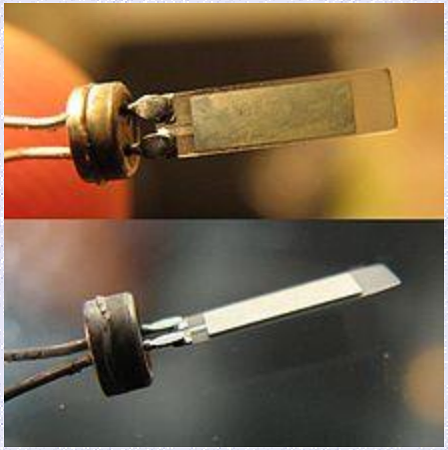


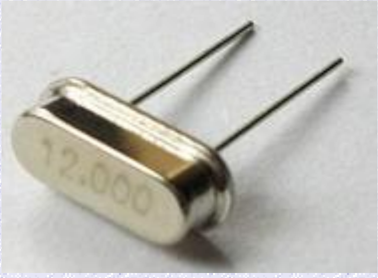
0,7 μs

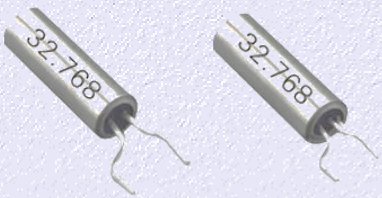


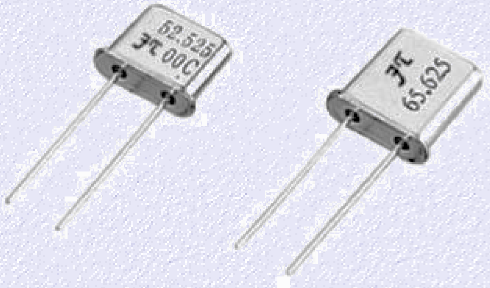
Quartz resonators

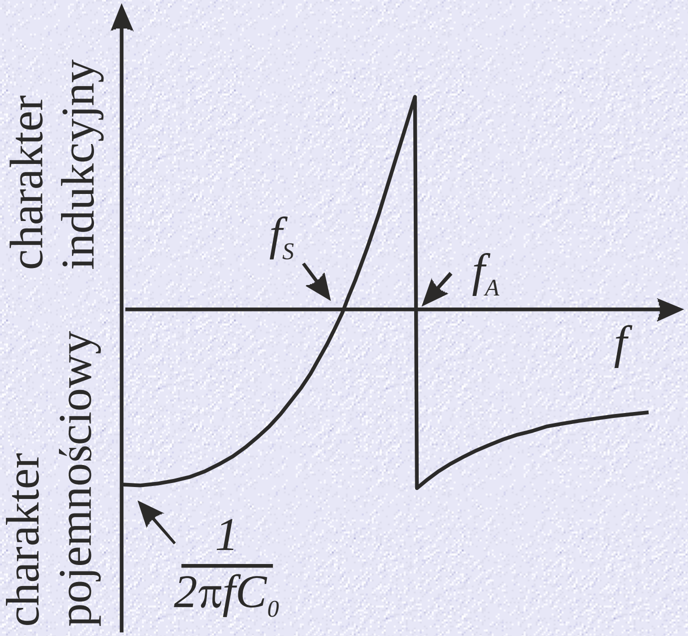


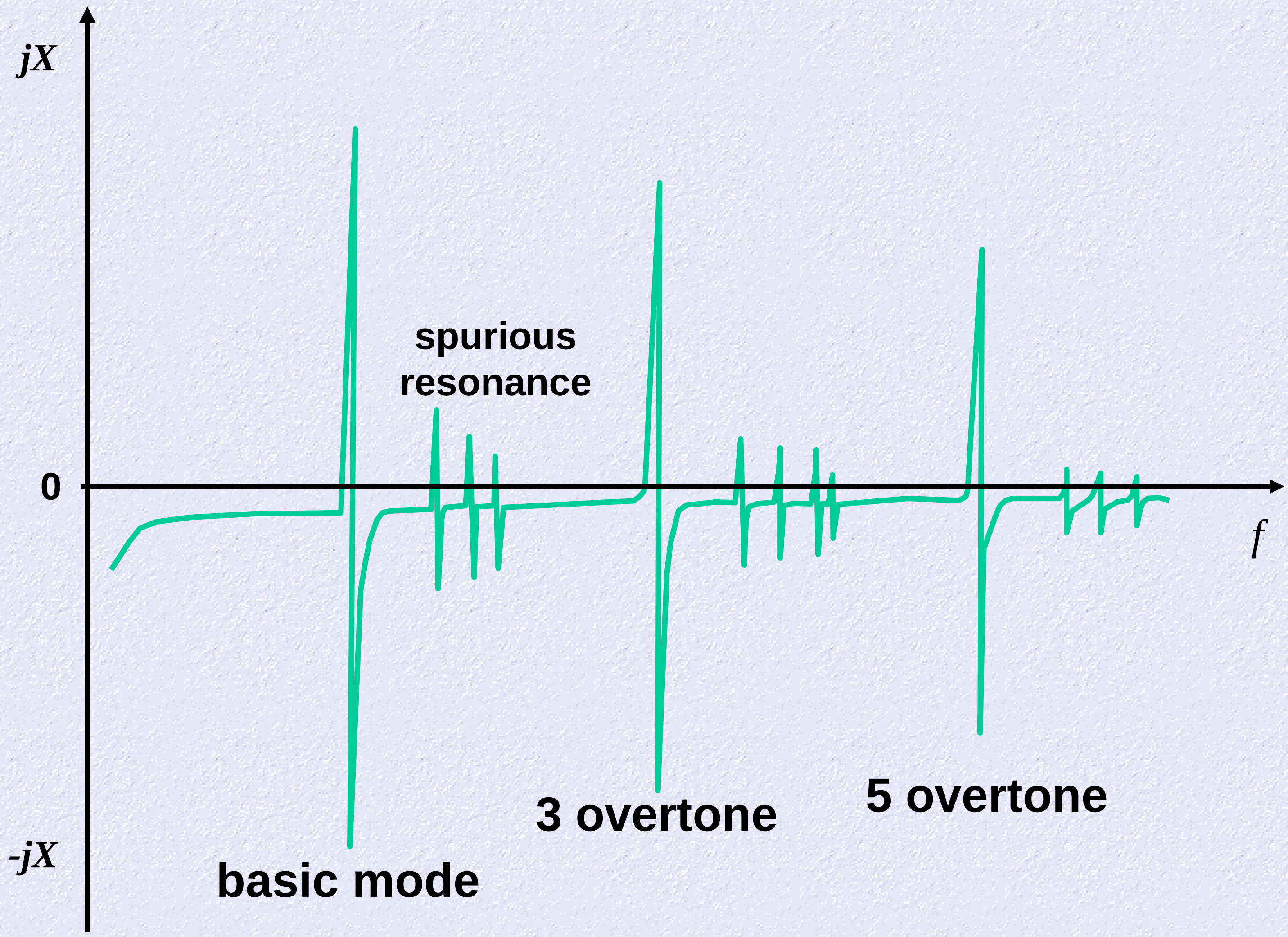




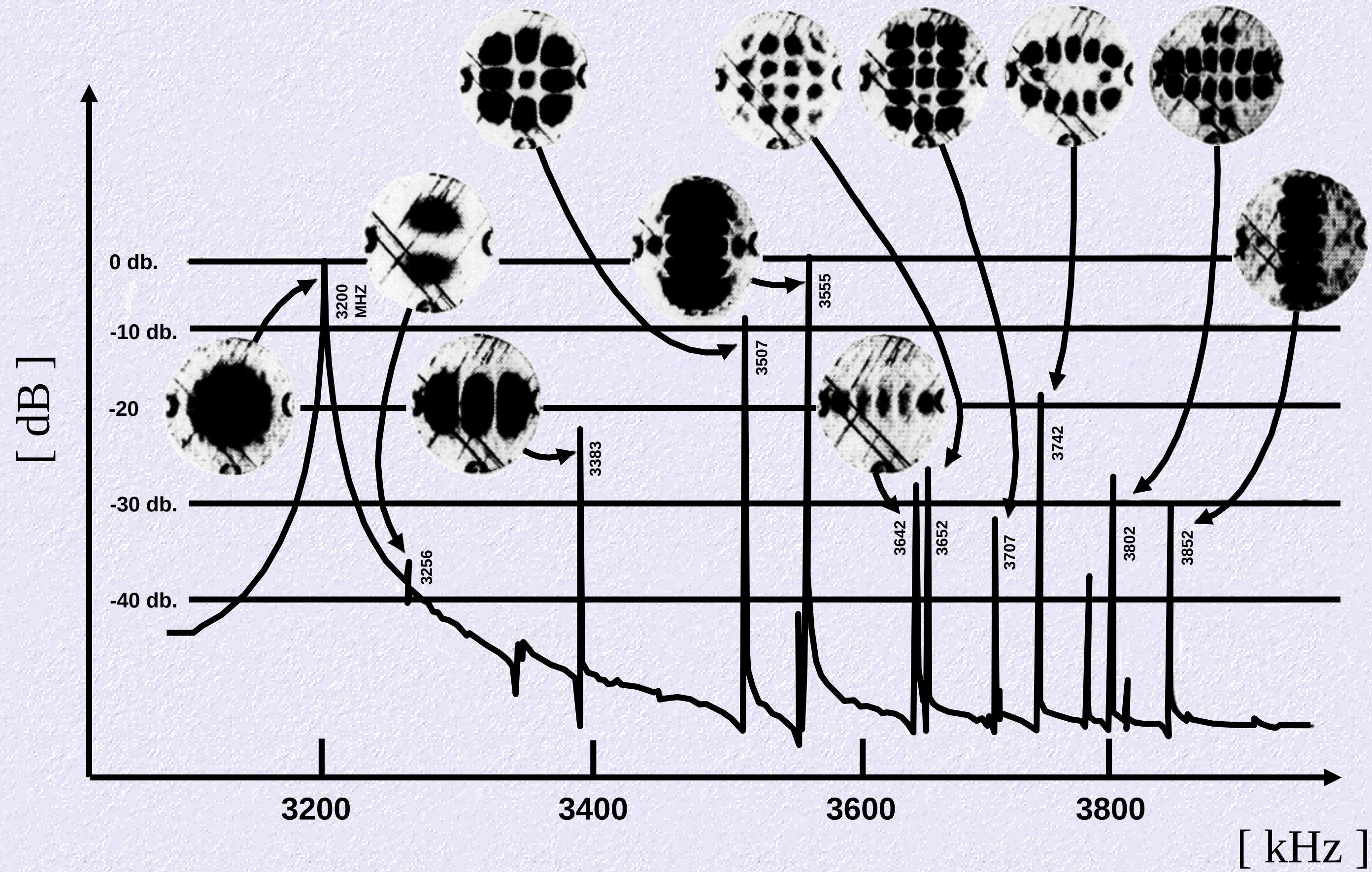




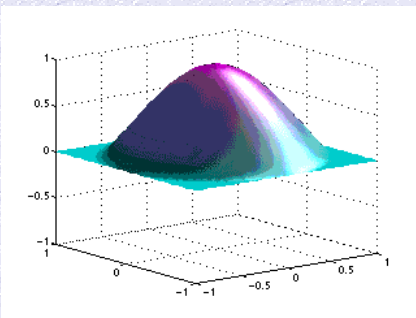


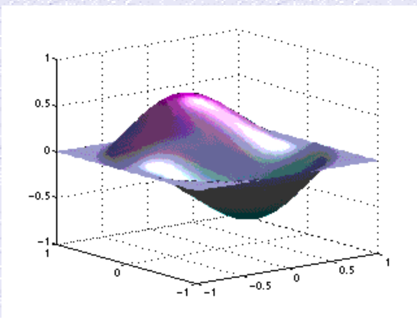


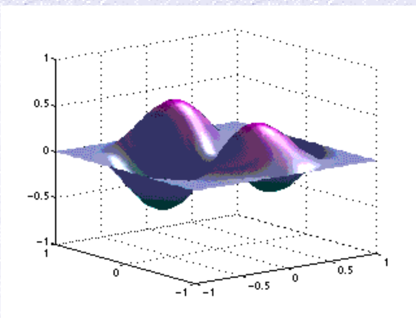
jX
spurious
resonance
0
f
3 overtone
5 overtone
-jX
basic mode

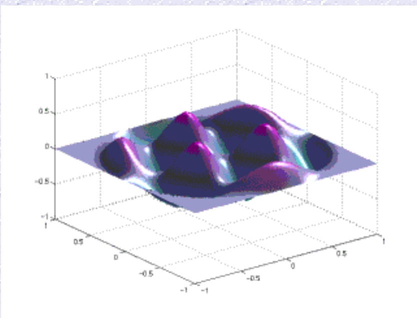


0 db.
-10 db.
-20
-30 db.
-40 db.
[dB]
3200 MHZ
3256
3383
3507
3555
3642
3652
3707
3742
3802
3852
3200
3400
3600
3800
[kHz]









Thin plate shear modes

Resonant frequency

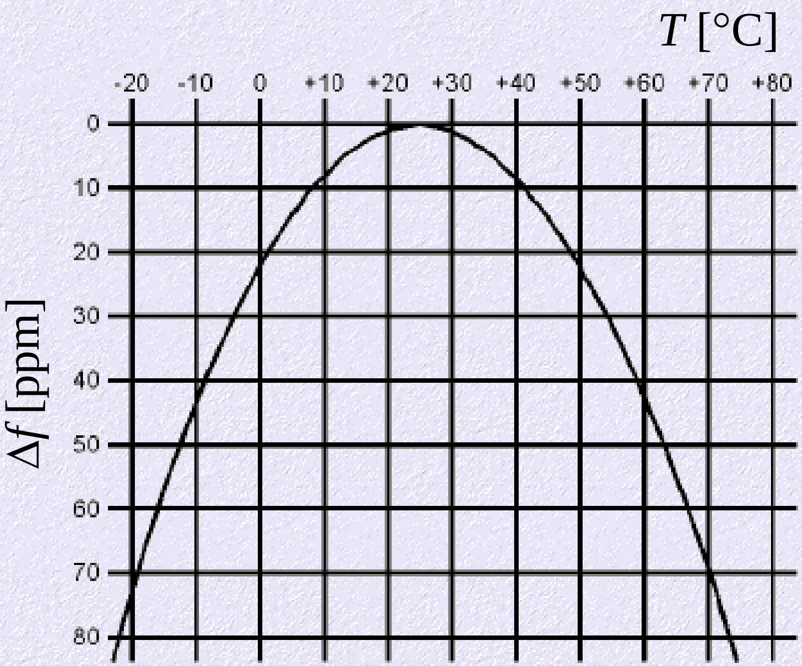
$$f_n = \frac{n}{2h}\sqrt{\frac{C_{ij}}{\rho}}, \quad n = 1, 3, 5...$$

f_n frequency of n-th overtone
h thickness
ρ density
C_{ij} elastic moduli

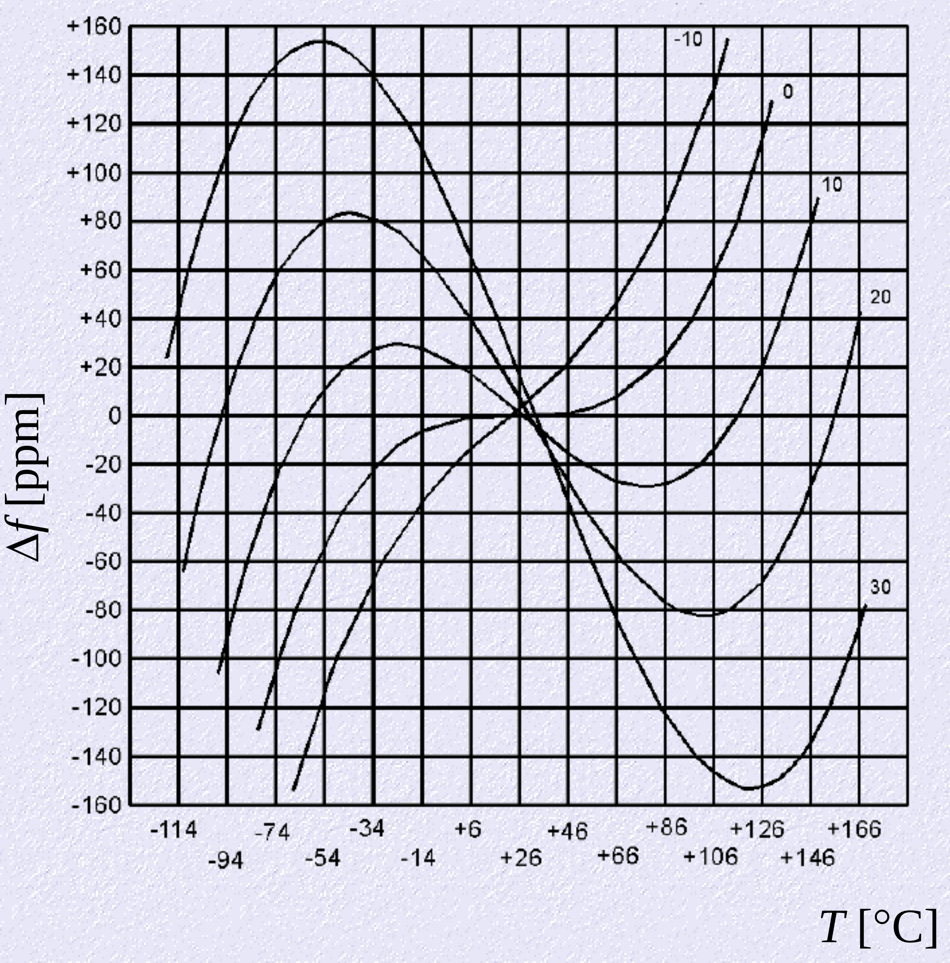
Linear temperature coefficient

$$T_f = \frac{d\left(\log f_n\right)}{dT} = \frac{1}{f_n}\frac{df_n}{dT} = \frac{-1}{h}\frac{dh}{dT} - \frac{1}{2\rho}\frac{d\rho}{dT} + \frac{1}{2c_{ij}}\frac{dc_{ij}}{dT}$$

Temperature dependences

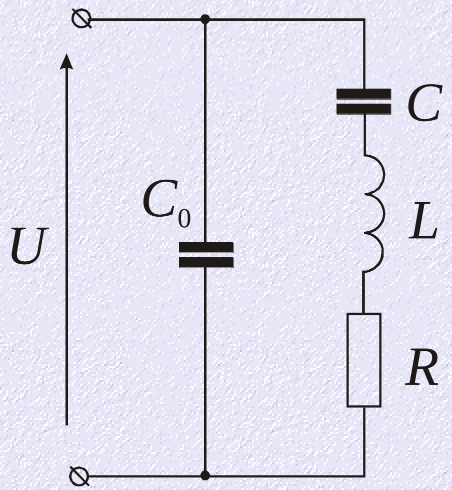


AT – cut –10° to +30°



materiał	cięcie	kierunek propagacji	prędkość fali Rayleigha [m/s]	współczynnik sprzężenia elektromech. [%]	współczynnik temperaturowy [10^{-6}/K]	względna przenikalność elektryczna
kwarc	42,75° Y (ST)	X	3175	0,16	0	4,5
kwarc	- 75° Y	X	3960	0,11	9	4,5
$LiNbO_3$	Y	Z	3488	4,82	-94	36,7
$LiNbO_3$	128° Y	X	4000	5,56	-74	39,1
$LiNbO_3$	64° Y	X	4742	11,3	-79	37,1
$LiTaO_3$	X	112° Y	3295	0,64	-18	44,0
$LiTaO_3$	36° Y	X	4178	4,8	-33	48,3
$Li_2B_4O_7$	45° X	Z	3401	1,0	0	9,6
polikryst. ZnO na szkle	-	-	2576	1,4	-11	10,8
kryst. ZnO na szafirze	-	-	5500	3,4	-7	10
$Pb(Sn_{1/2}Sb_{1/2})O_3$ $+PbTiO_3+PbZrO_3$	-	-	2420	2,4	-38	270
$0{,}1Pb(Mn_{1/3}Nb_{2/3})O_3$ $+0{,}9Pb(Zr_{0,74}Ti_{0,26})O_3$	-	-	2430	2,9	-17	460

Equivalent circuit – more details



$$C_0 \cong \varepsilon \frac{A}{h} \qquad r = \frac{C_0}{C}$$

$$f_S = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$f_a - f_s \approx \frac{f_s}{2r} \qquad Q = \frac{1}{2\pi f_s RC}$$

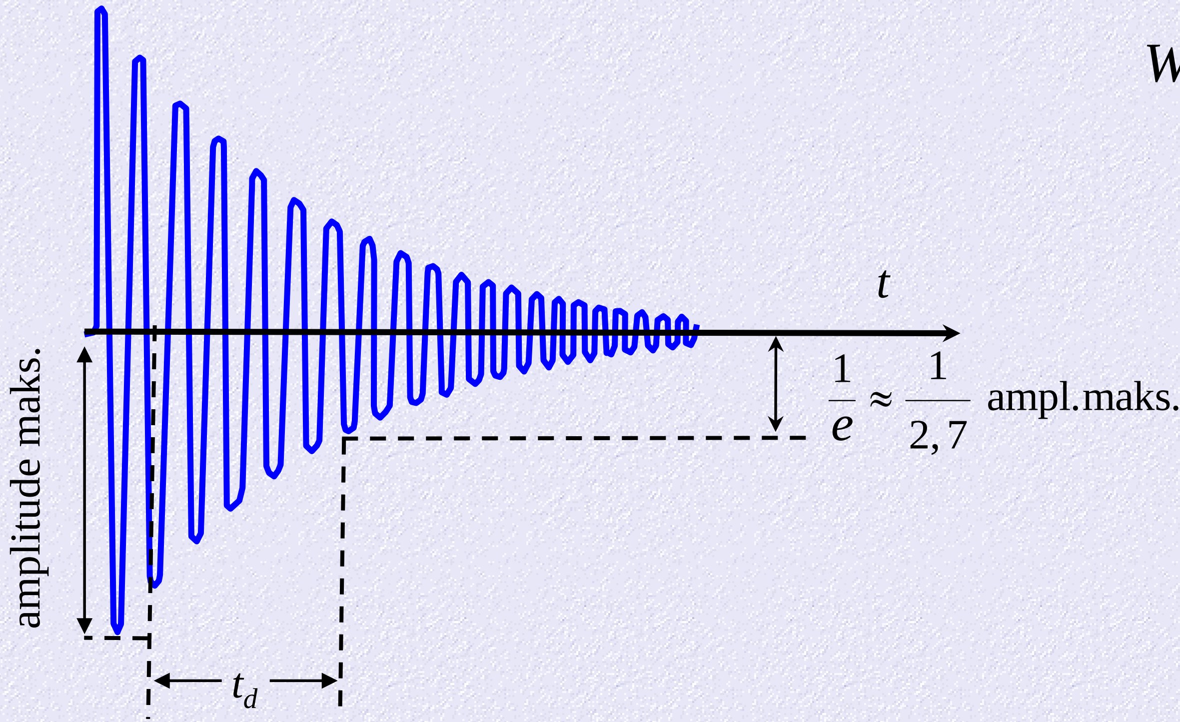
n overtone number
C_n capacity
L_n inductance
R_n resistance
ε permittivity for quartz ≈ 40 pF/m
A area of electrodes
h thickness of plate
r capacitive coeff.
f_s frequency of serial resonance
f_a antiresonance frequency
Q Q-factor
τ group delay
φ phase shift
K electromechanical coupling coeff.

$$\varphi = \frac{\omega L - \frac{1}{\omega C}}{R} \qquad \tau = RC \approx 10^{-14}\mathrm{s} \qquad \frac{d\varphi}{df} \cong \frac{360}{\pi}\frac{Q}{f_s}$$

$$C_n \approx \frac{r' C_{11}}{n^3} \qquad L_n \approx \frac{n^3 L_{11}}{r'^3} \qquad R_n \approx \frac{n^3 R_{11}}{r'} \qquad r' = \frac{f_1}{f_n} \qquad 2r = \left(\frac{\pi n}{2K}\right)^2$$

Q factor

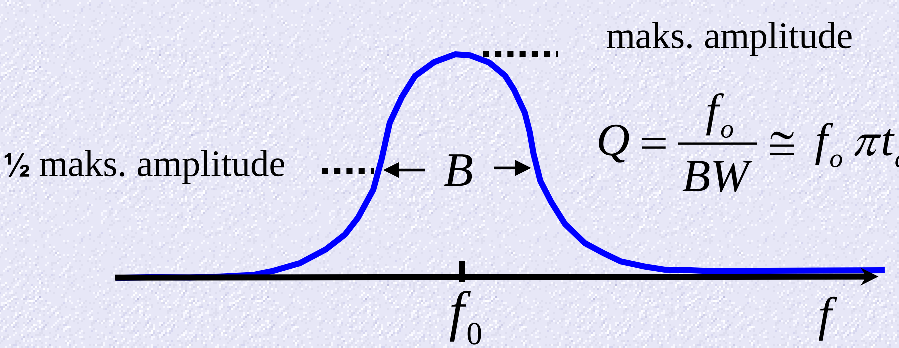
$$W(t) = W_0 \exp\left(-\frac{\omega_0 t}{Q}\right)$$



Power loss in time unit

$$P = -\frac{dW}{dt}$$

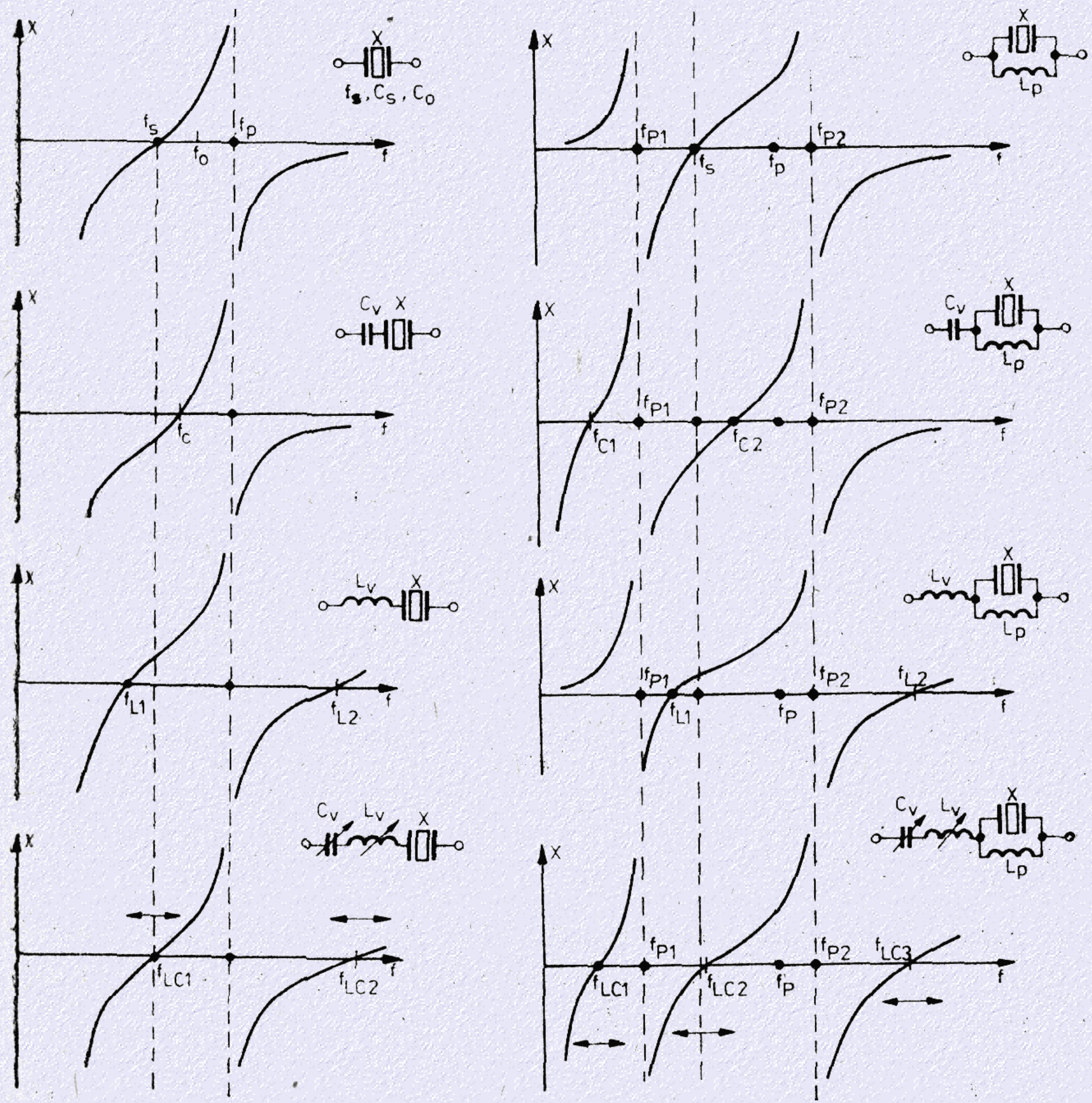
$$Q = \omega_0 \frac{W_0}{P}$$



$$Q = \frac{f_o}{BW} \cong f_o \pi t_d$$

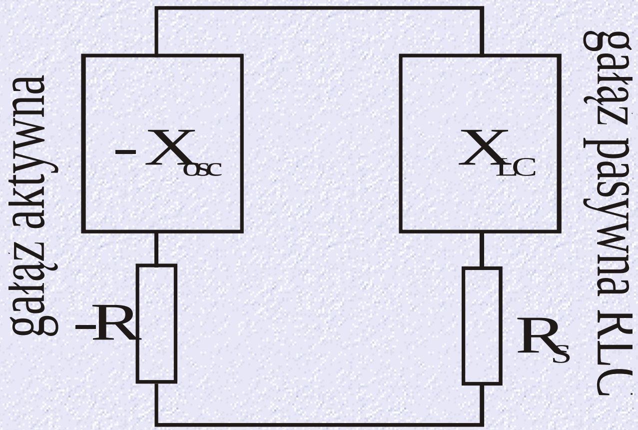
$$B \cong \frac{1}{\pi t_d}$$

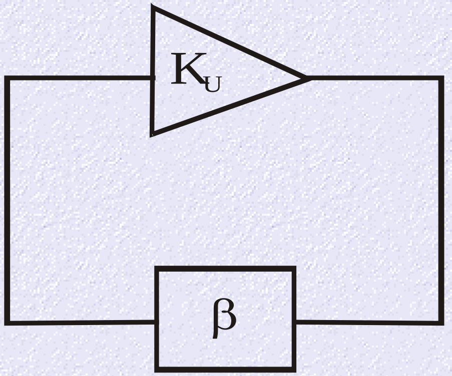
Resonator dragging



Quartz stabilized oscillators

XO - Crystal Oscillator
VCXO - Voltage Controlled Crystal Oscillator
OCXO - Oven Controlled Crystal Oscillator
TCXO - Temperature Compensated Crystal Oscillator
TCVCXO - Temperature Compensated/Voltage Controlled Crystal Oscillator
OCVCXO - Oven Controlled/Voltage Controlled Crystal Oscillator
MCXO - Microcomputer Compensated Crystal Oscillator
RbXO - Rubidium-Crystal Oscillator

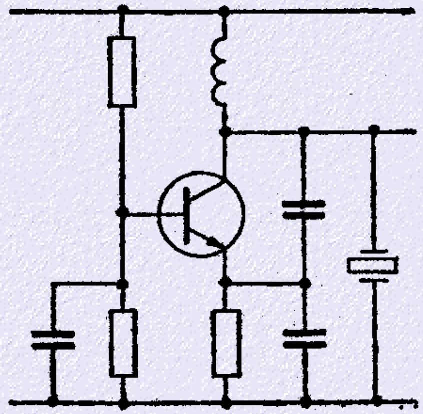


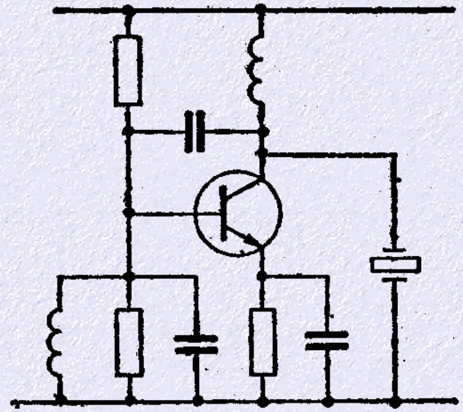


Warunki oscylacji!

Basic circuits

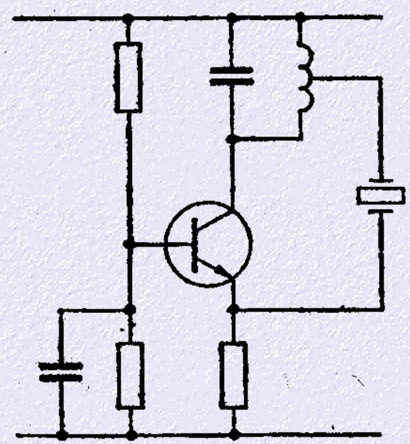
Colpittsa

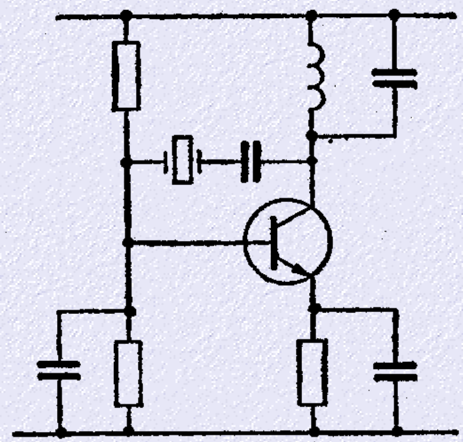




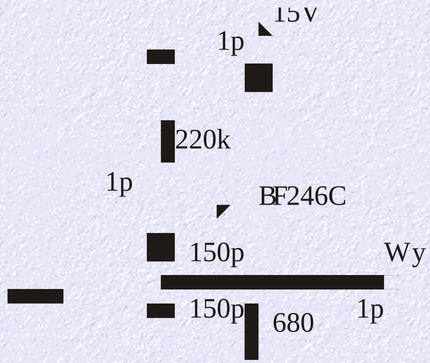
rezonansowy

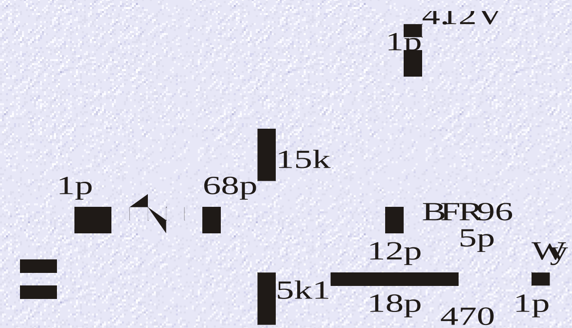
Hartleya

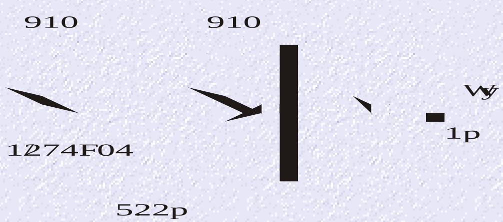




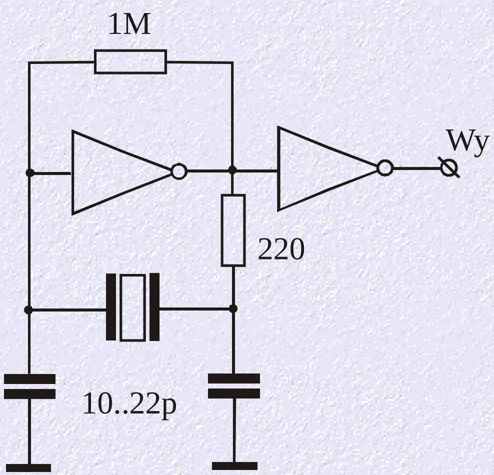
Pierce’a



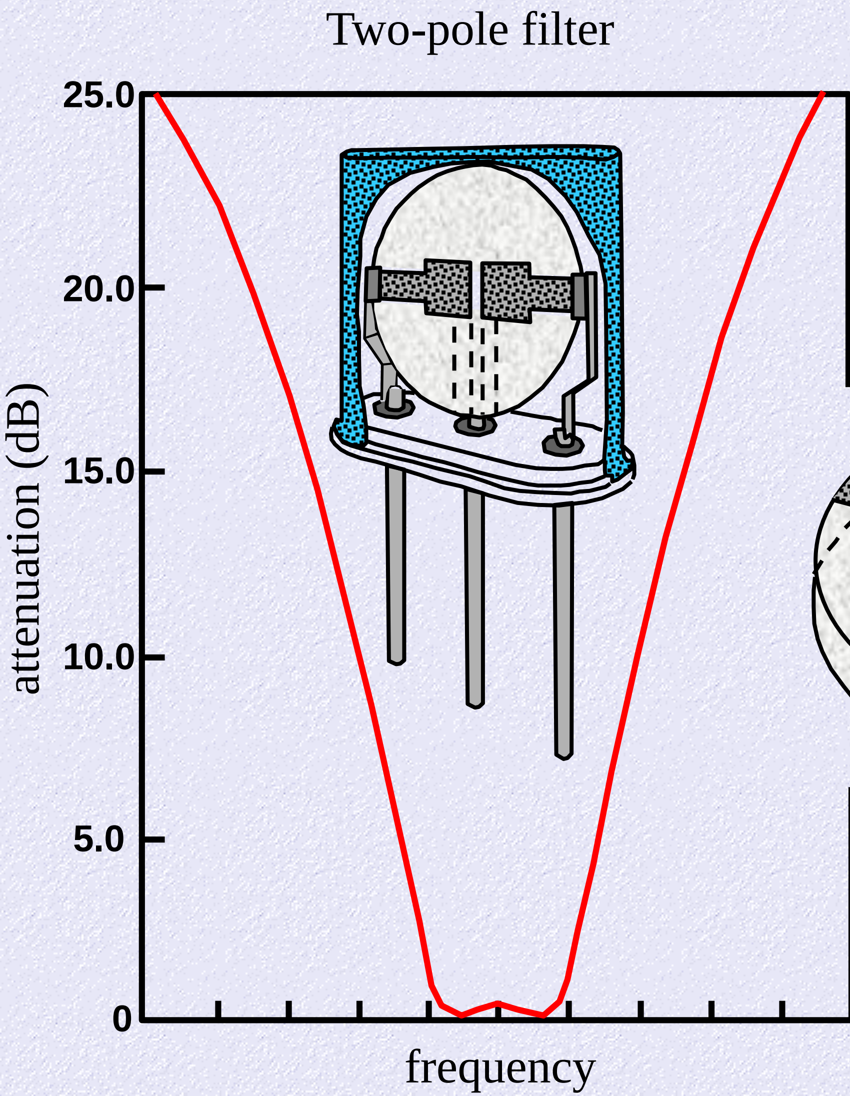




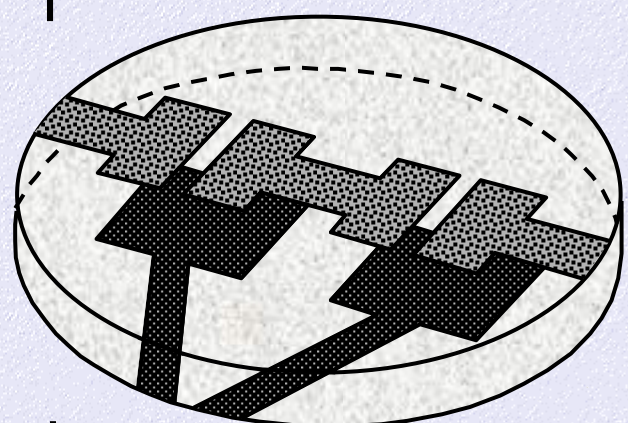
Overtone circuit

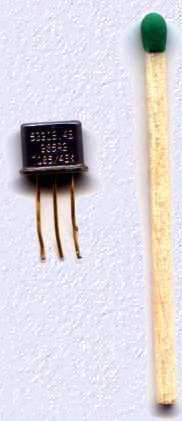


BAW filters



4-pole BAW filter

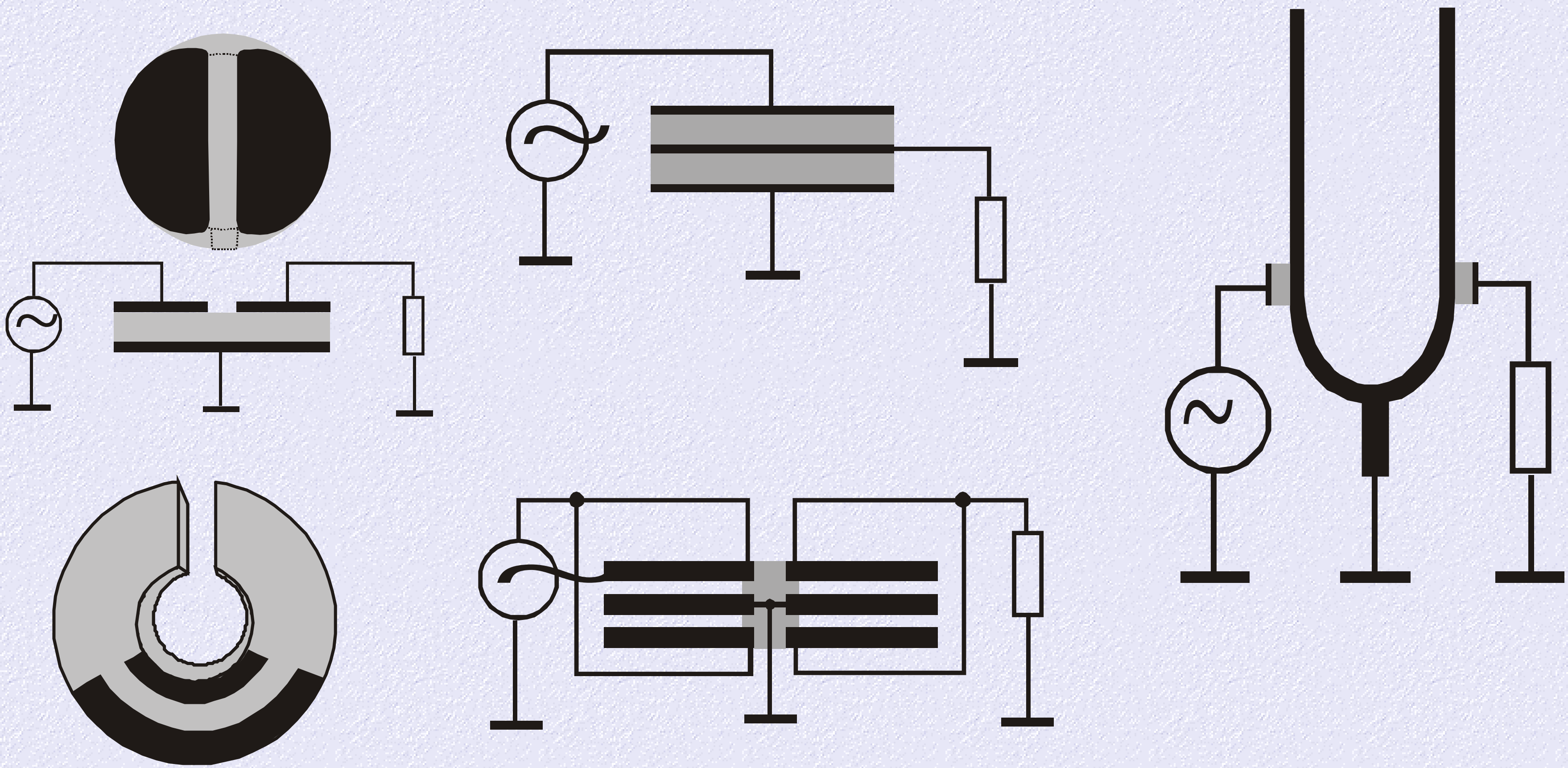


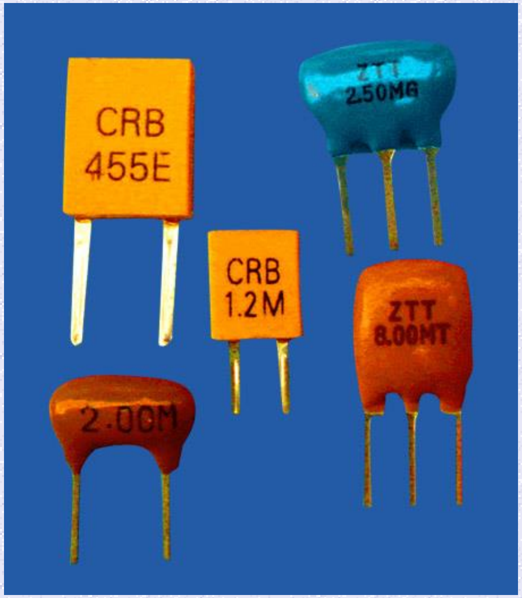


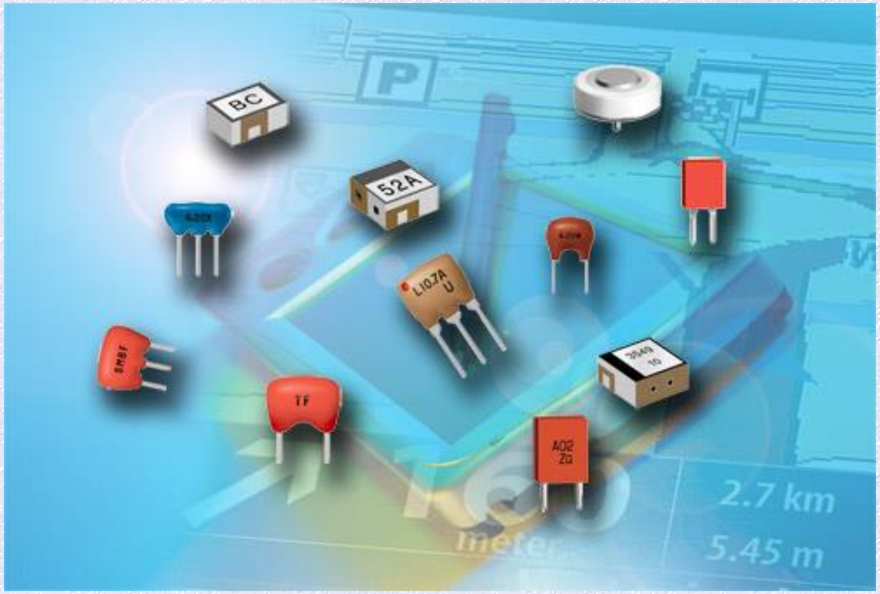
BAW filters

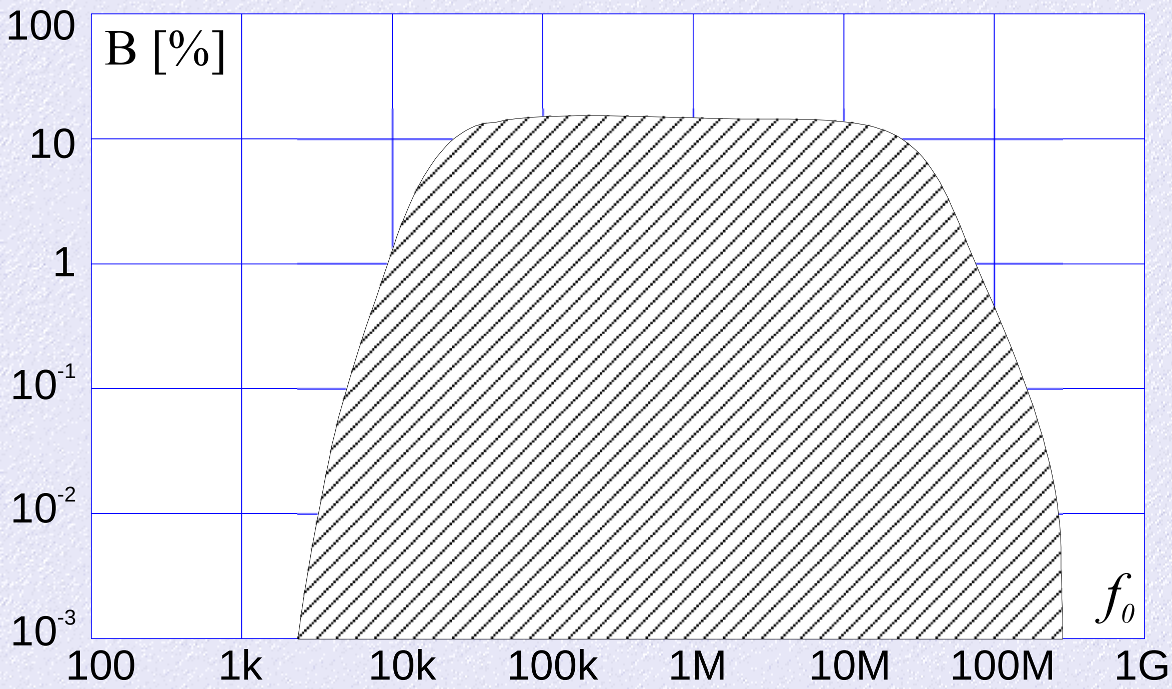
Two-pole | multipole

planar | bimorphic | camertone | ring

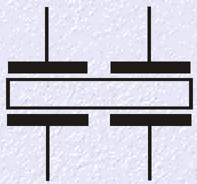



CRB
455E
ZTT
2.50MG
CRB
1.2M
ZTT
8.00MT
2.00M

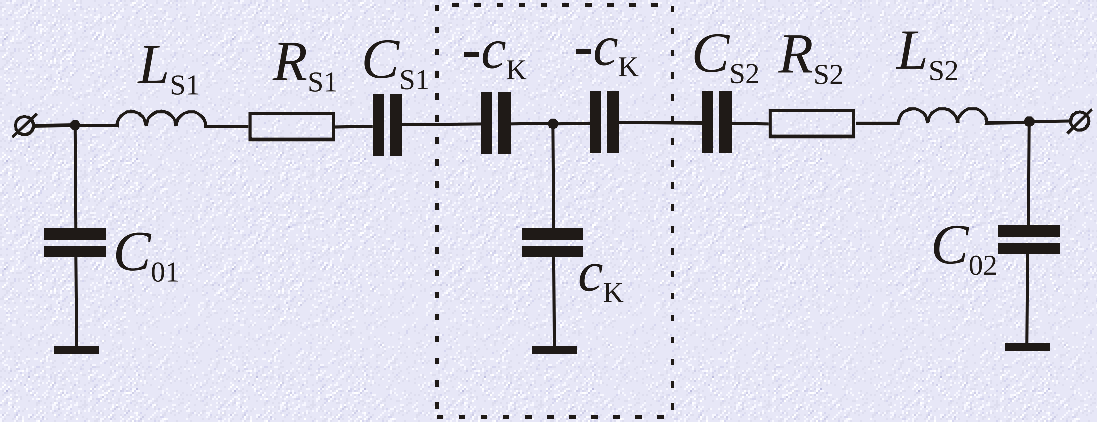

BC
52A
TF
2.7 km
5.45 m


B [%]
100
10
1
10^{-1}
10^{-2}
10^{-3}
f_0
100
1k
10k
100k
1M
10M
100M
1G

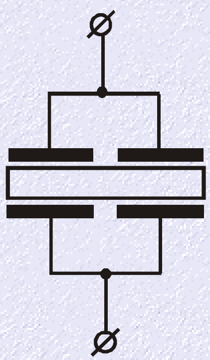
symbol



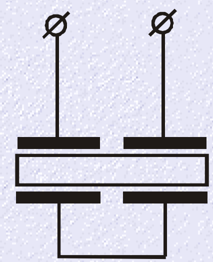
Equivalent circuit



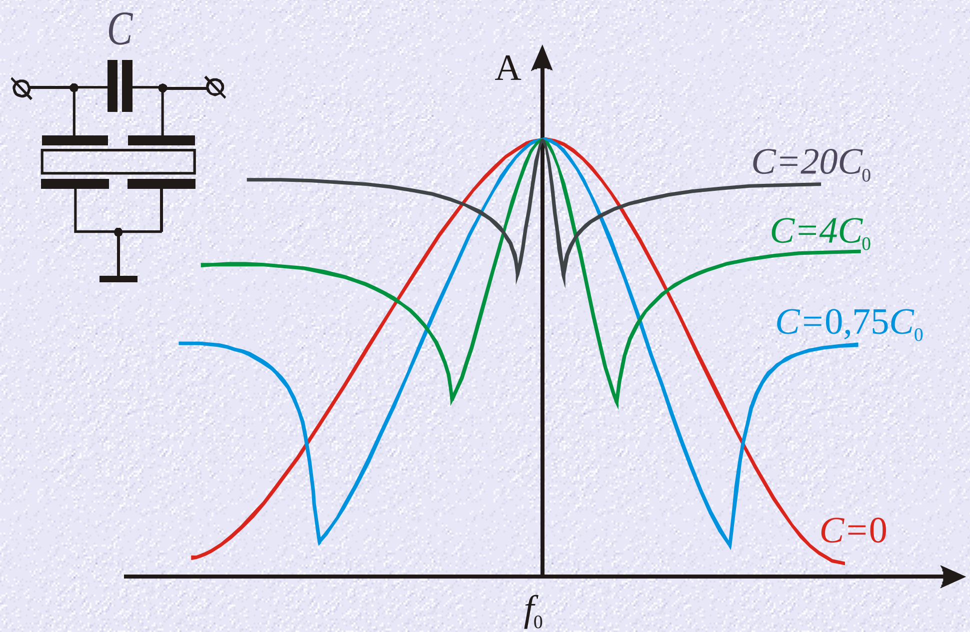
Methods of connection



symmetric



asymmetric



For most common two – pole filter resonant frequencies of input and output part are equal.

$$f_{1,2} = f_S \sqrt{1 + \frac{C_S}{2C_0} \pm \sqrt{K^2 + \left(\frac{C_S}{2C_0}\right)^2}}$$

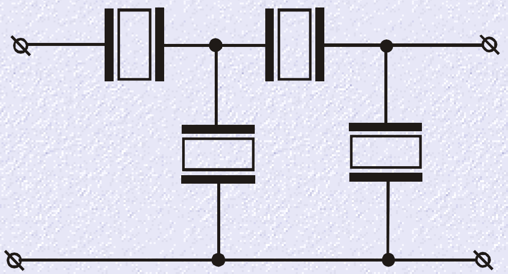
$K = \dfrac{C_S}{c_K}$ electromechanical coupling coeff.

for symmetric configuration $f_{SYM} = f_S \sqrt{1 - K}$ $f_{SYM} < f_S$

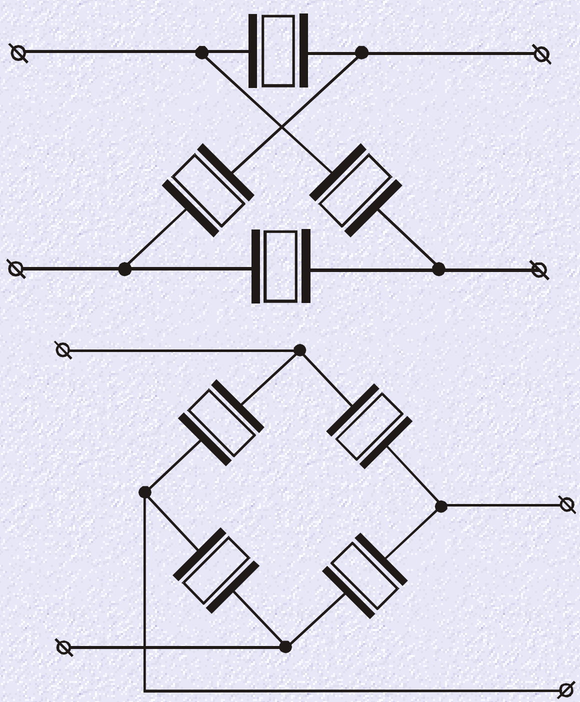
for asymmetric configuration $f_{ASYM} = f_S \sqrt{1 + K} \cong f_{SYM}(1 + K)$ $f_{ASYM} > f_S$

$$f_{ASYM} - f_{SYM} = f_{SYM} K$$

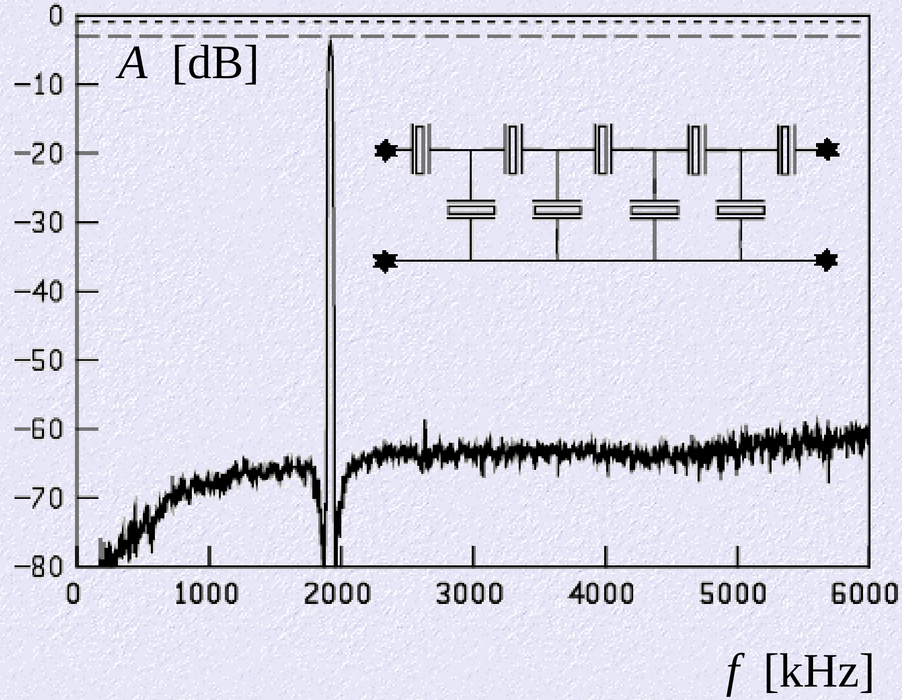
Multichain configurations

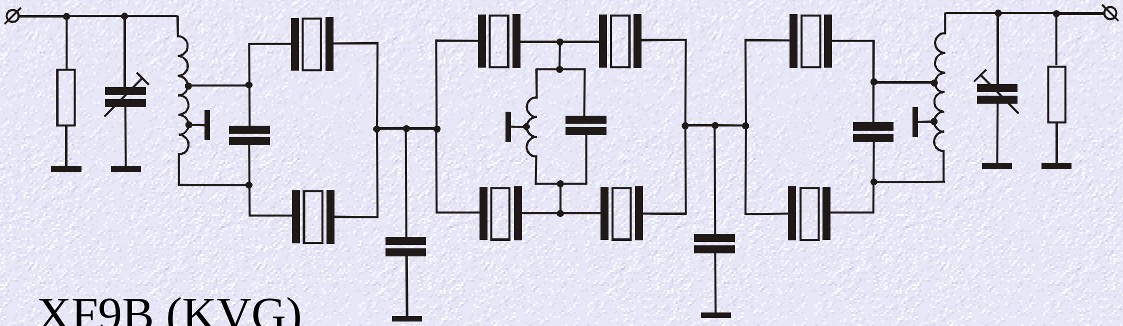


ladder

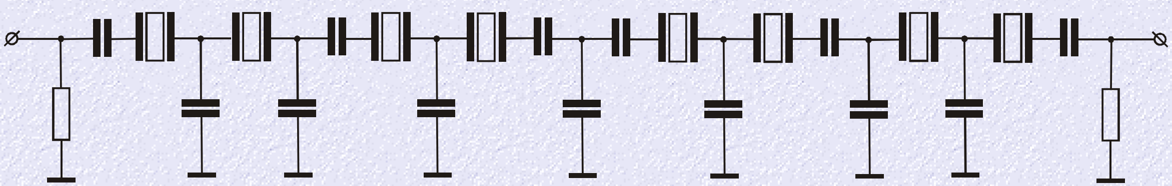


bridge

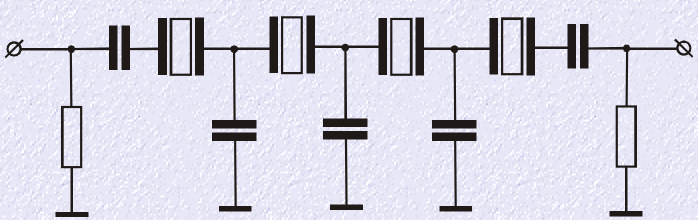




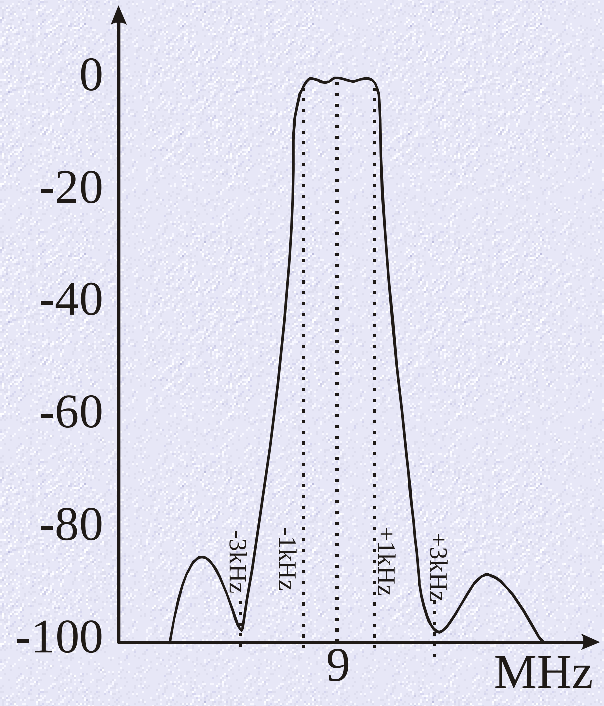
XF9B (KVG)

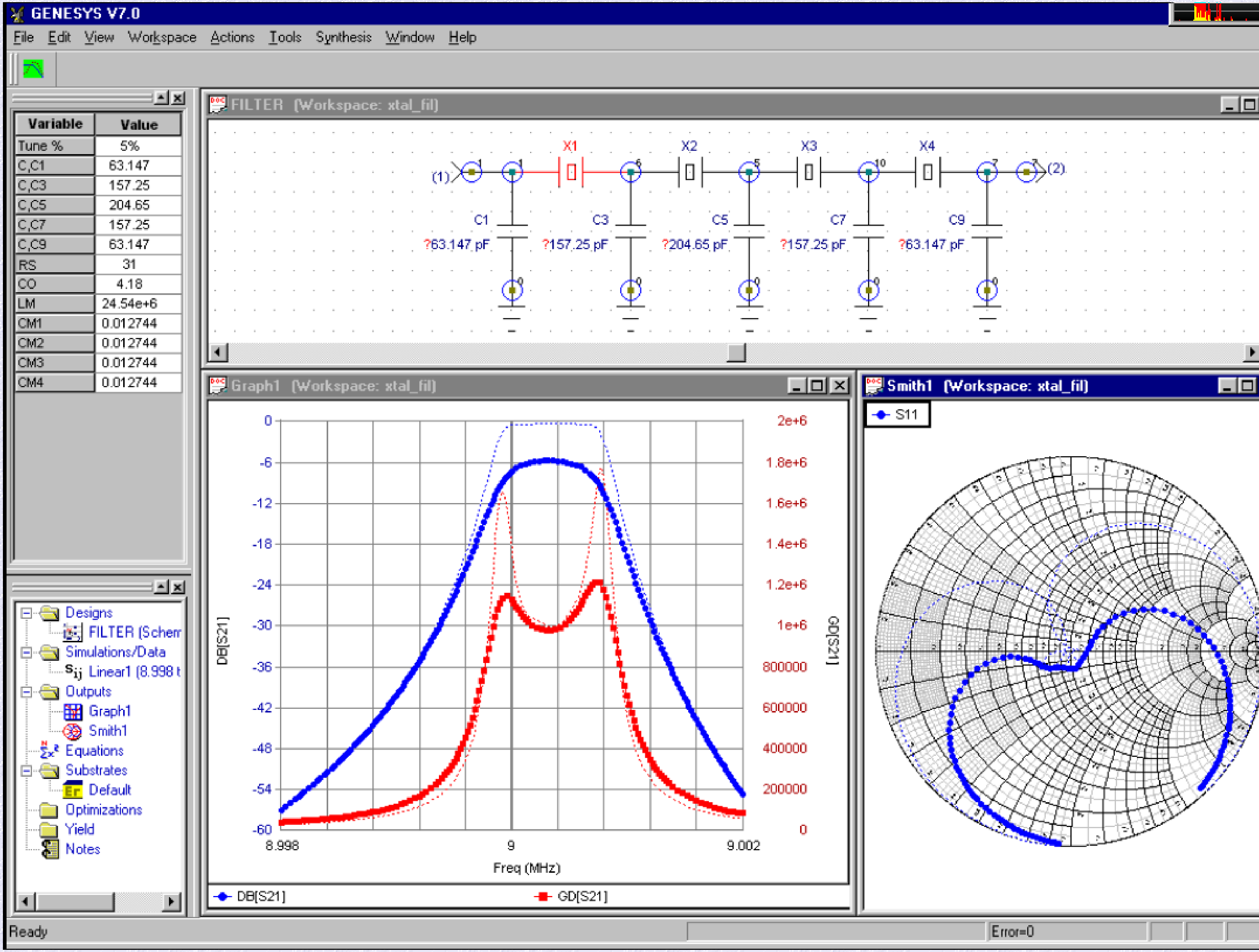


Czybyszew type filter



Butterworth type filter




GENESYS V7.0
File Edit View Workspace Actions Tools Synthesis Window Help
Variable Value
Tune % 5%
C,C1 63.147
C,C3 157.25
C,C5 204.65
C,C7 157.25
C,C9 63.147
RS 31
CO 4.18
LM 24.54e+6
CM1 0.012744
CM2 0.012744
CM3 0.012744
CM4 0.012744
Designs
FILTER (Schem
Simulations/Data
Linear1 (8.998 t
Outputs
Graph1
Smith1
Equations
Substrates
Default
Optimizations
Yield
Notes
FILTER (Workspace: xtal_fil)
X1
X2
X3
X4
(1)
(2)
C1
C3
C5
C7
C9
?63.147 pF
?157.25 pF
?204.65 pF
?157.25 pF
?63.147 pF
Graph1 (Workspace: xtal_fil)
DB[S21]
GD[S21]
Freq (MHz)
8.998
9
9.002
Smith1 (Workspace: xtal_fil)
S11
Ready
Error=0

SAW excitation and devices

WHY?

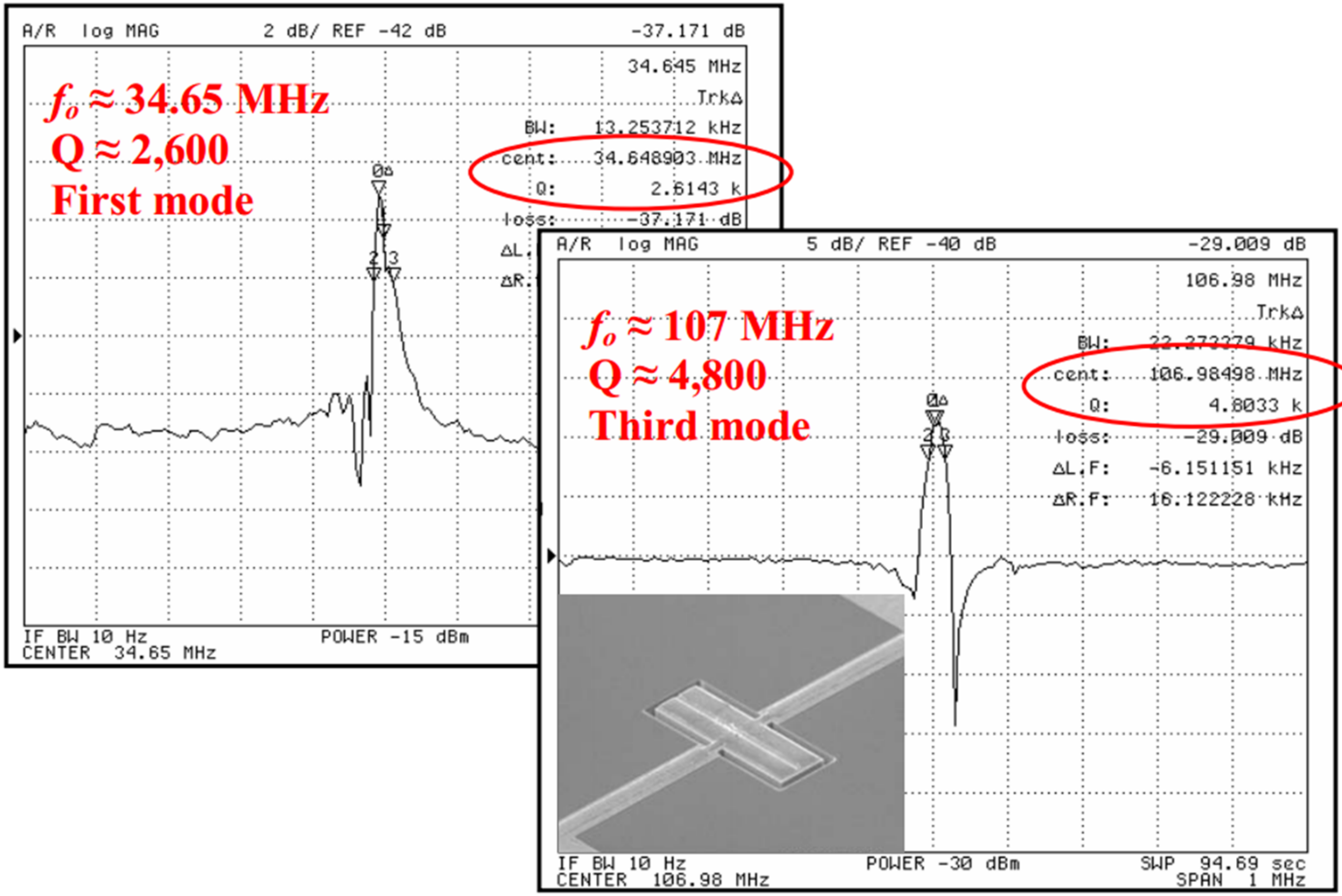
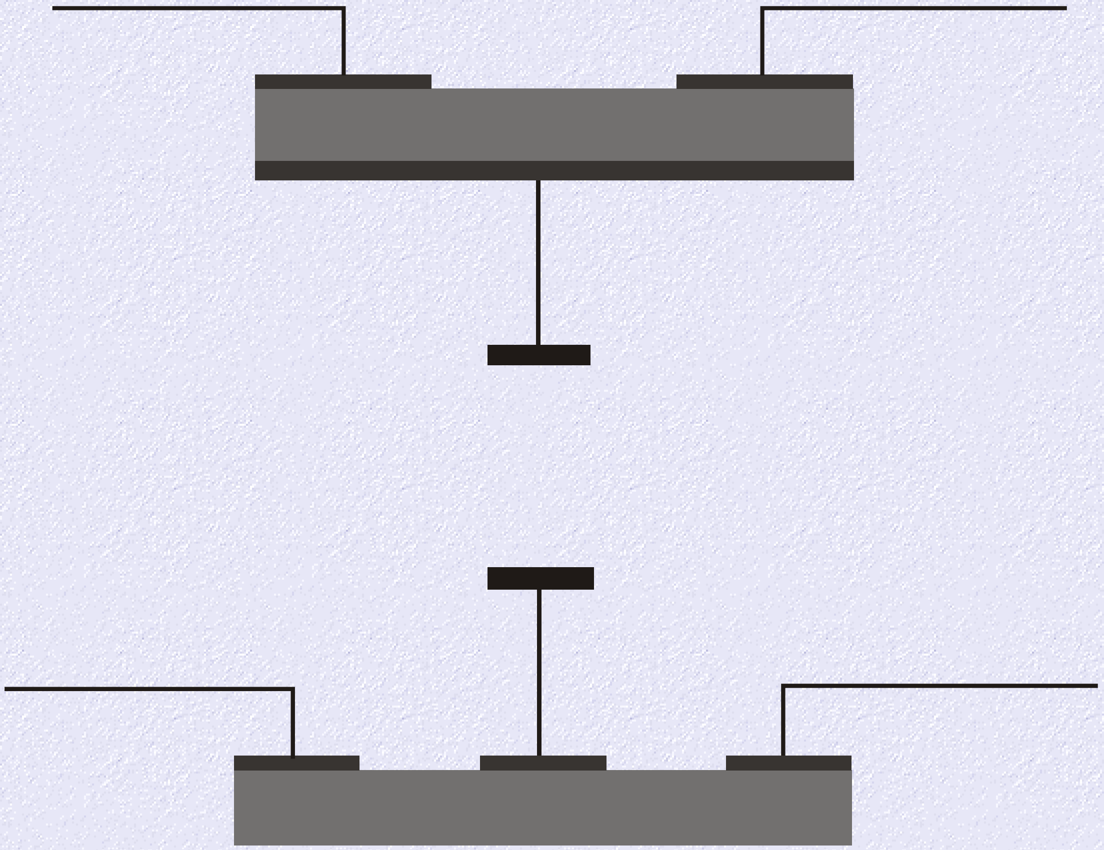
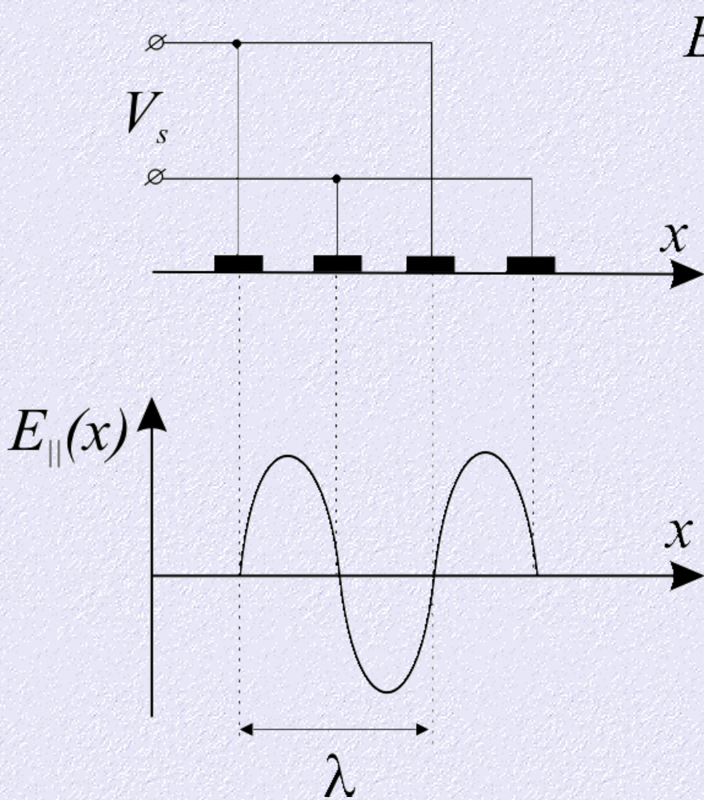


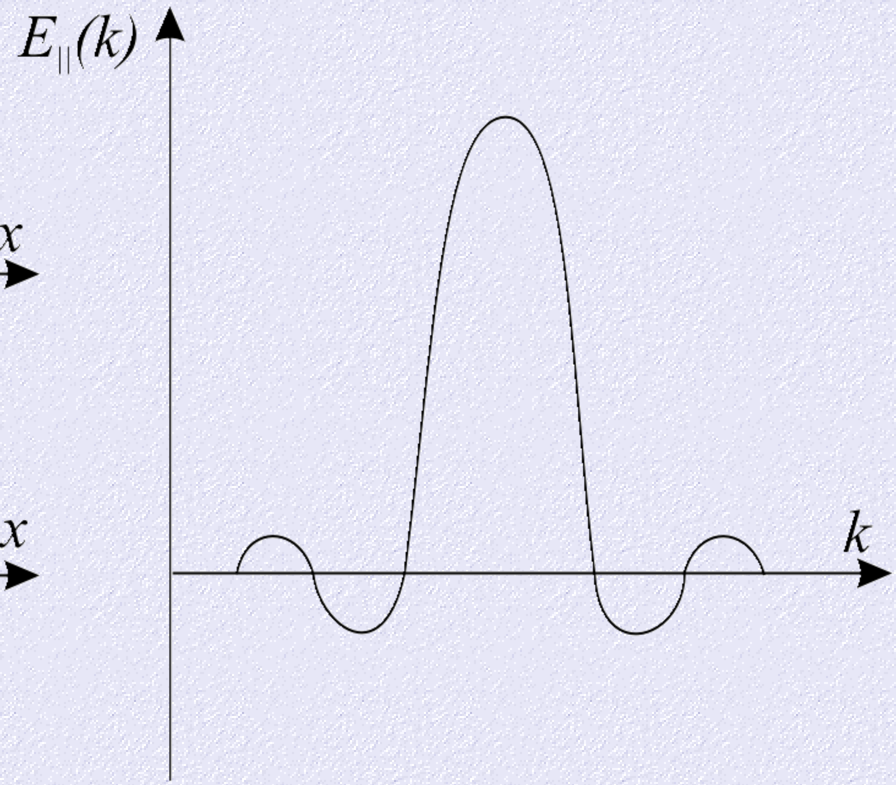
Fig. 7 First and third high-Q length extensional modes measured at 34.65MHz and 107MHz for a 120μm×30μm piezo-on-silicon block.

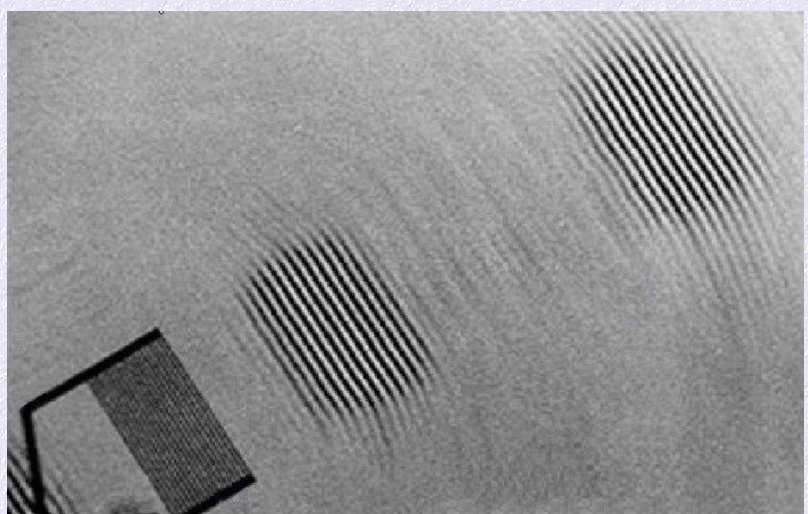
Evolution from BAW to SAW - from two planar electrodes to interdigital transducer (IDT)



[illegible]


V_s
x
$E_{||}(x)$
x
λ

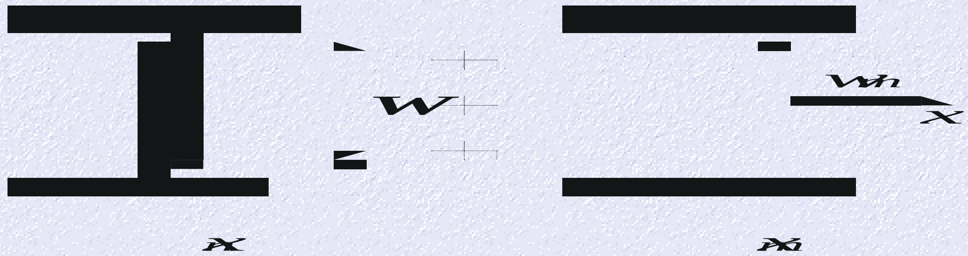

$E_{||}(k)$
k



$V = \delta$ - Diraca

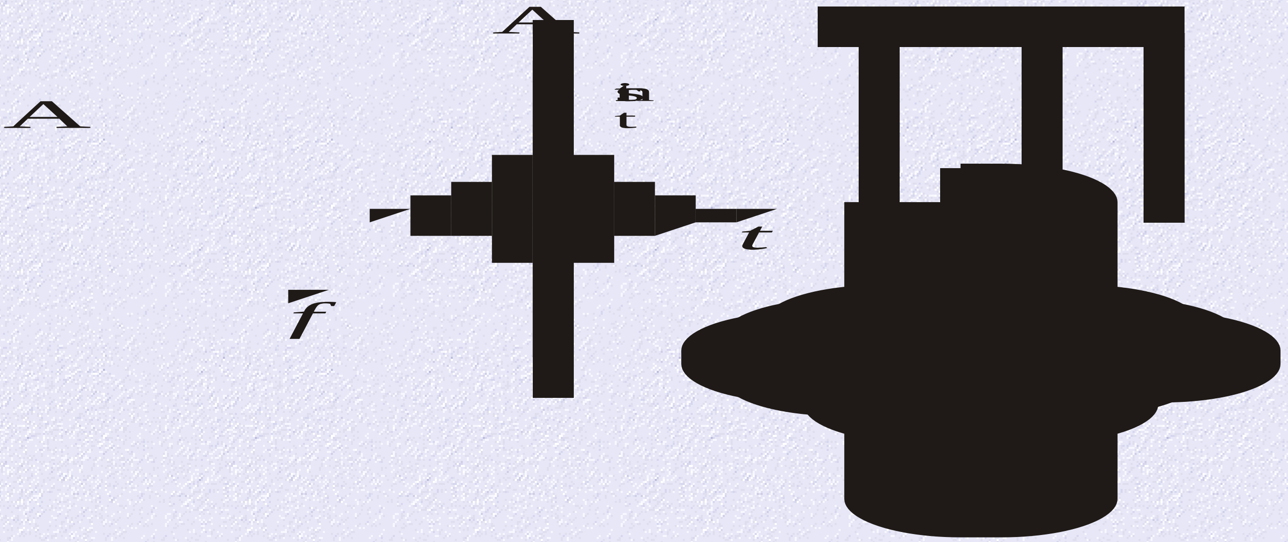
$$A(\omega) \approx \sum E_n e^{-j\omega x_n / v}$$

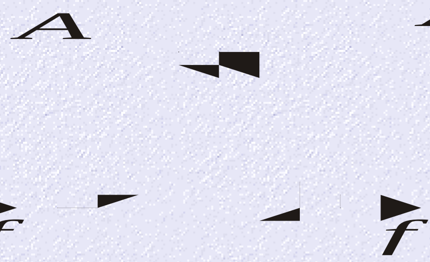
E_n amplitudes of Dirac pulses between electrodes in x_n

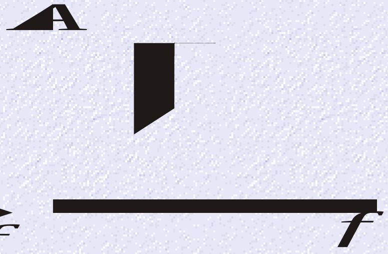


$$S(\tau) = s_1(\tau) * s_2(\tau) \qquad H(\omega) = H_1(\omega) H_2(\omega)$$

$$H(\omega) \sim \sum_{n,m} E_n e^{-j\omega x_n / v} w_m e^{j\omega x_m / v}$$


A
A
t
f

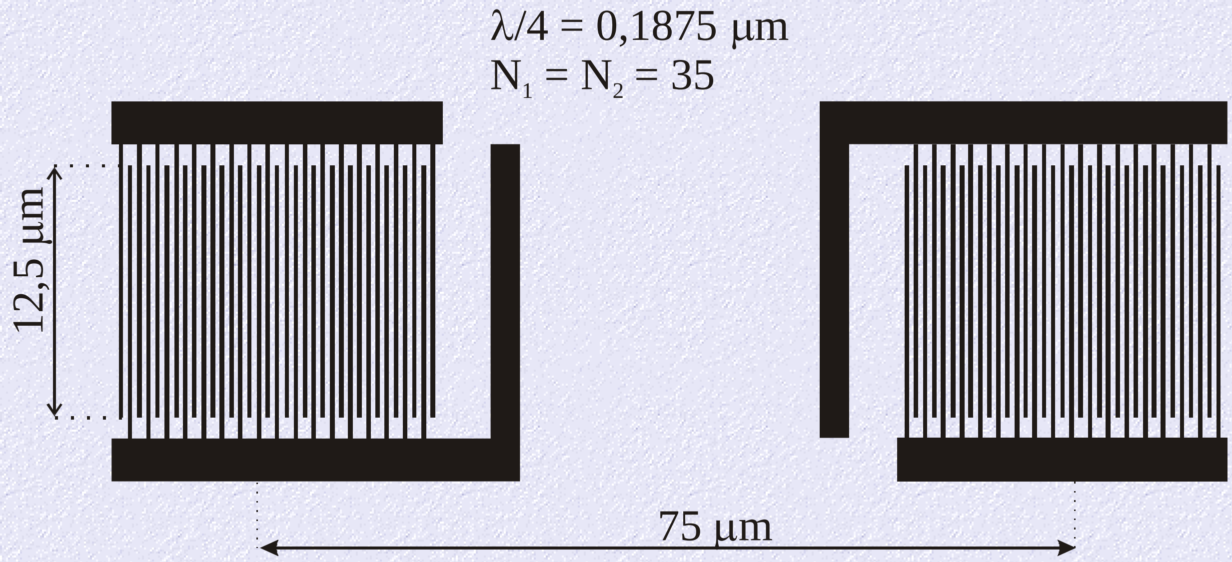


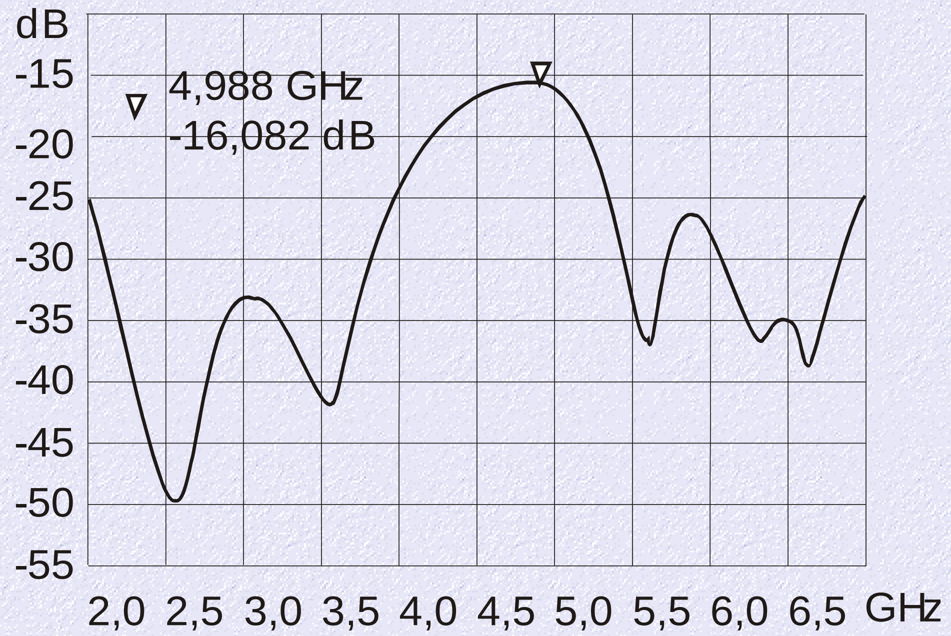


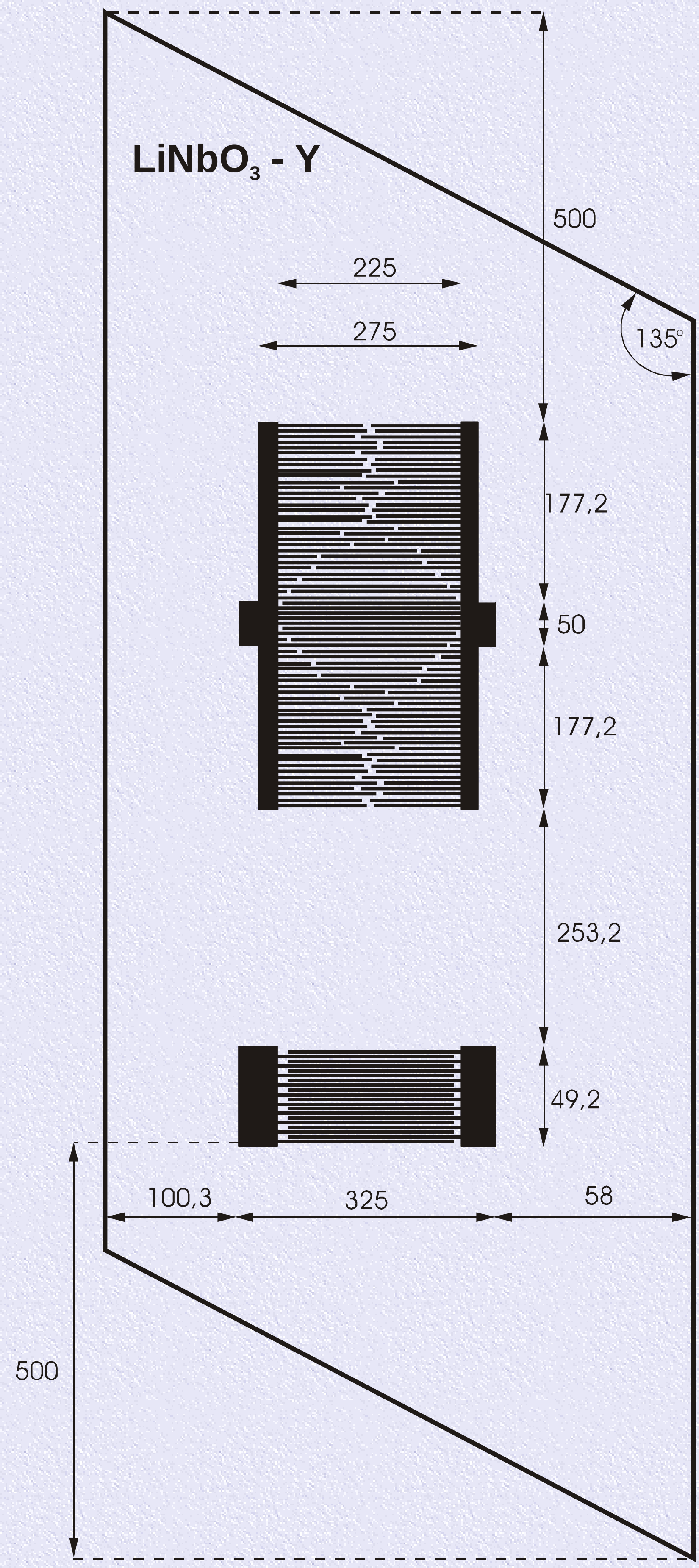
$$W = const$$

$$W = Sinc\left(x\right)F_{OP}$$

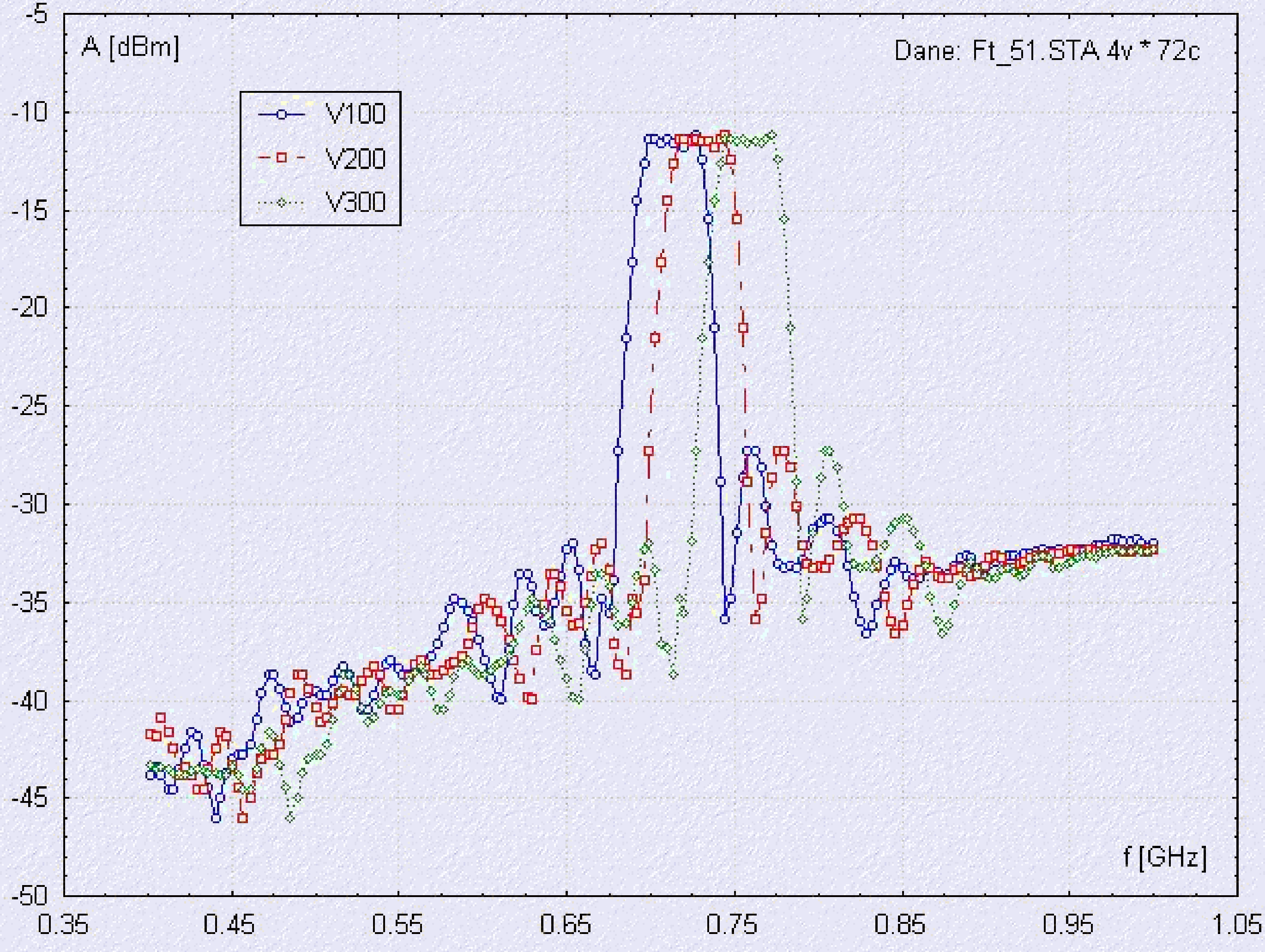
$$W = Sinc\left(x\right)F_{OP}F_{w}$$


λ/4 = 0,1875 μm
N1 = N2 = 35
12,5 μm
75 μm


dB
4,988 GHz
-16,082 dB
-15
-20
-25
-30
-35
-40
-45
-50
-55
2,0 2,5 3,0 3,5 4,0 4,5 5,0 5,5 6,0 6,5 GHz



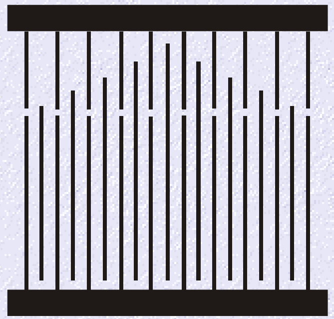
LiNbO3 - Y
500
225
275
135°
177,2
50
177,2
253,2
49,2
100,3
325
58
500


A [dBm]
Dane: Ft_51.STA 4v * 72c
V100
V200
V300
f [GHz]
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
0.35
0.45
0.55
0.65
0.75
0.85
0.95
1.05

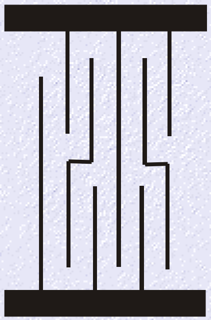
To improve parameters

- special weighting
- special constructions of IDT
- dummy electrodes
- splitted electrodes
- precise modelling and technology
- nonconventional substrates
- dumping of unwanted reflections
- application of different kind of waves
- directional couplers
- electromagnetic screening

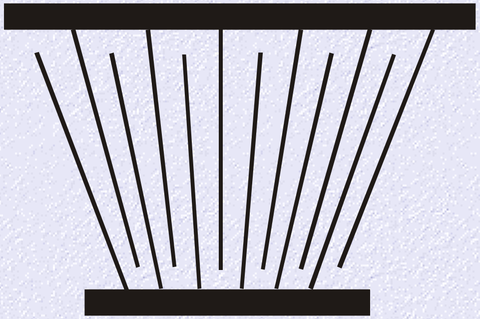
Common waighting methods



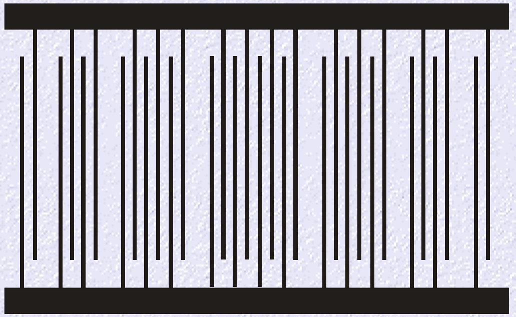
capacitive



broken electrodes

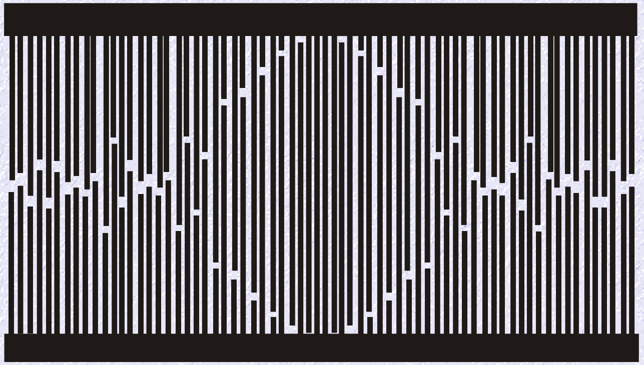


slanted electrodes

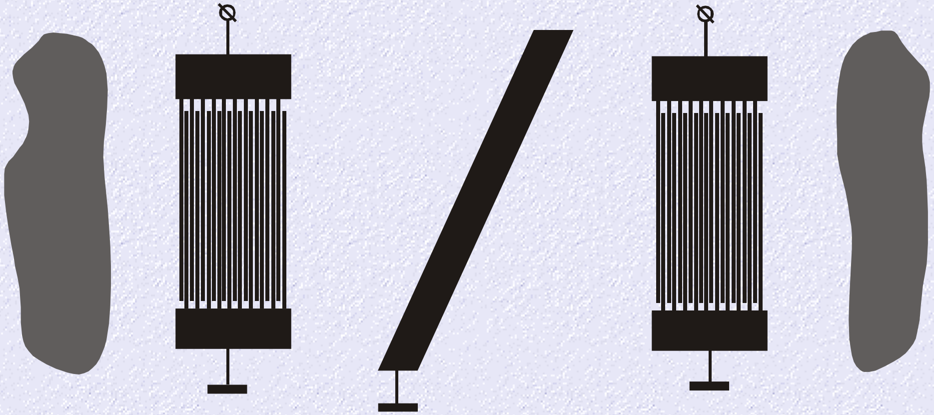


removed electrodes

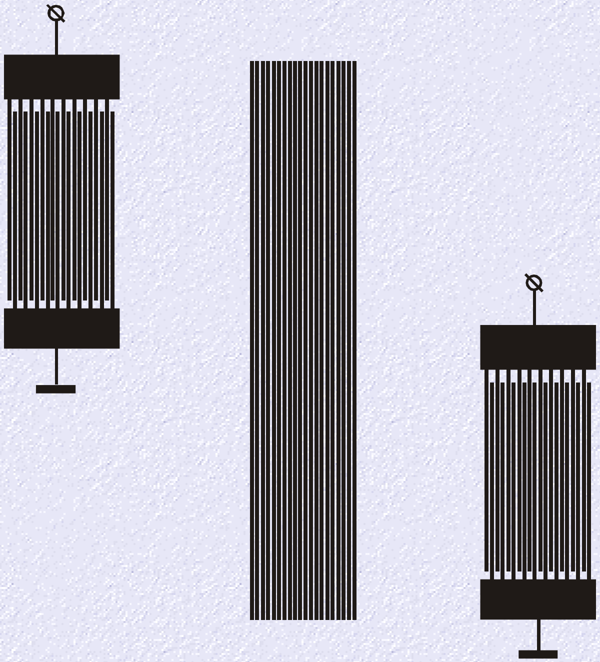
Methods of enchancement



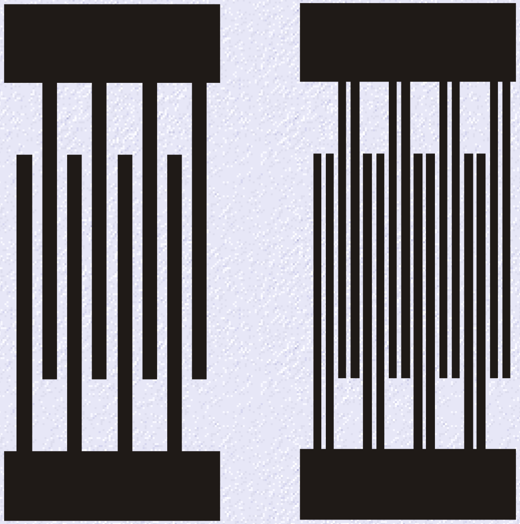
Dummy electrodes



Screening and dumping of reflections



directional couplers

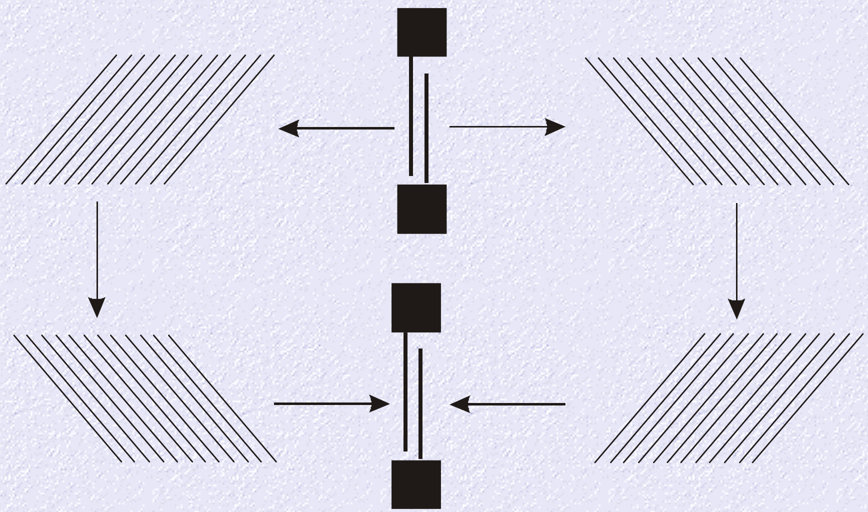


splitted electrodes

Using SAW one can obtain band-pass filters having

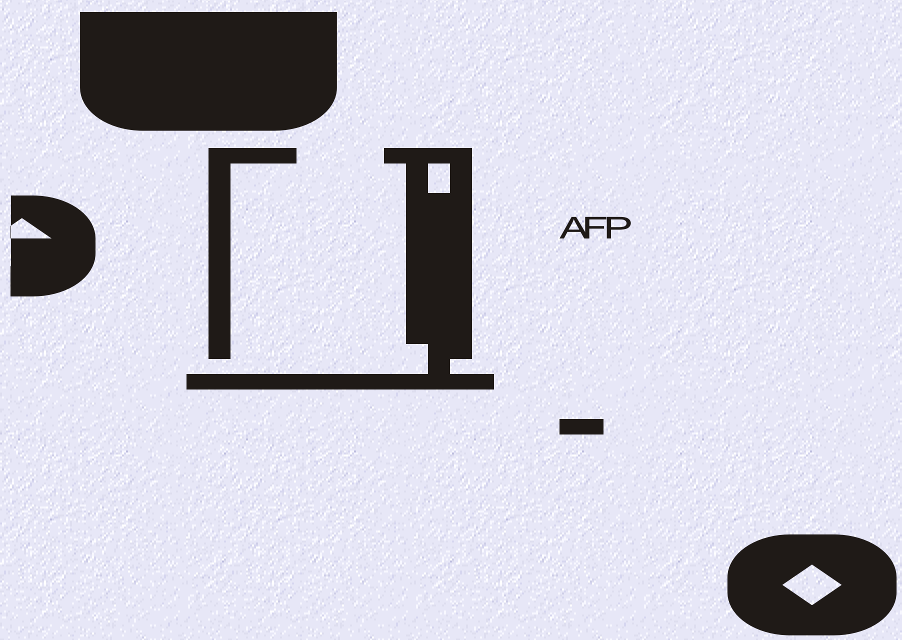
- 80 dB attenuation out of the bandwith
- few dB inside band
- small band-pass ripples ~0.1%
- precise characteristic – almost rectangular shape
- small sizes (the higher frequency the smaller)
- simple technology
- high repetability
- high time stability
- low cost

How to eliminate bidirectivity

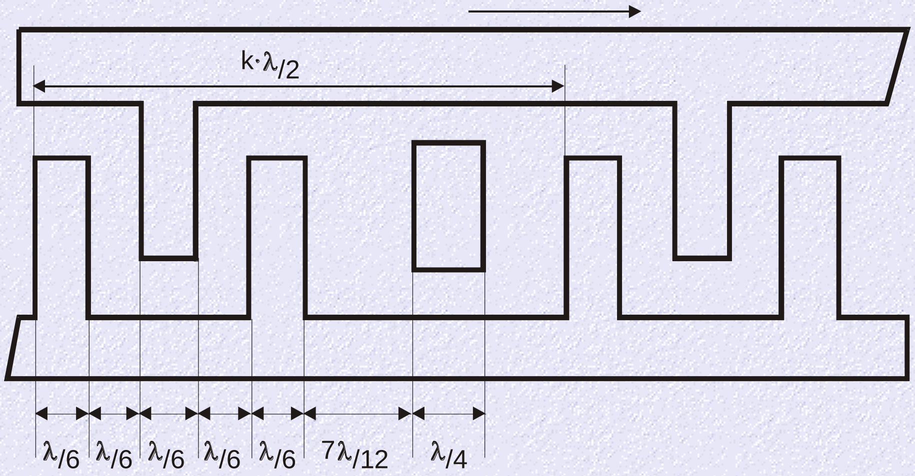


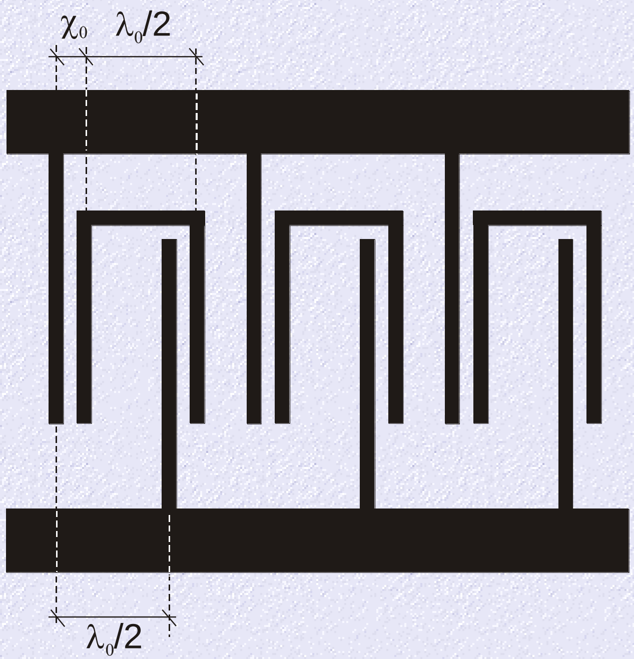
Reflectors application

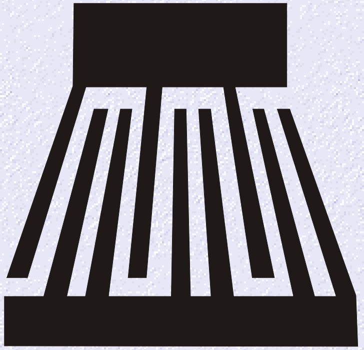
2-phase transducer

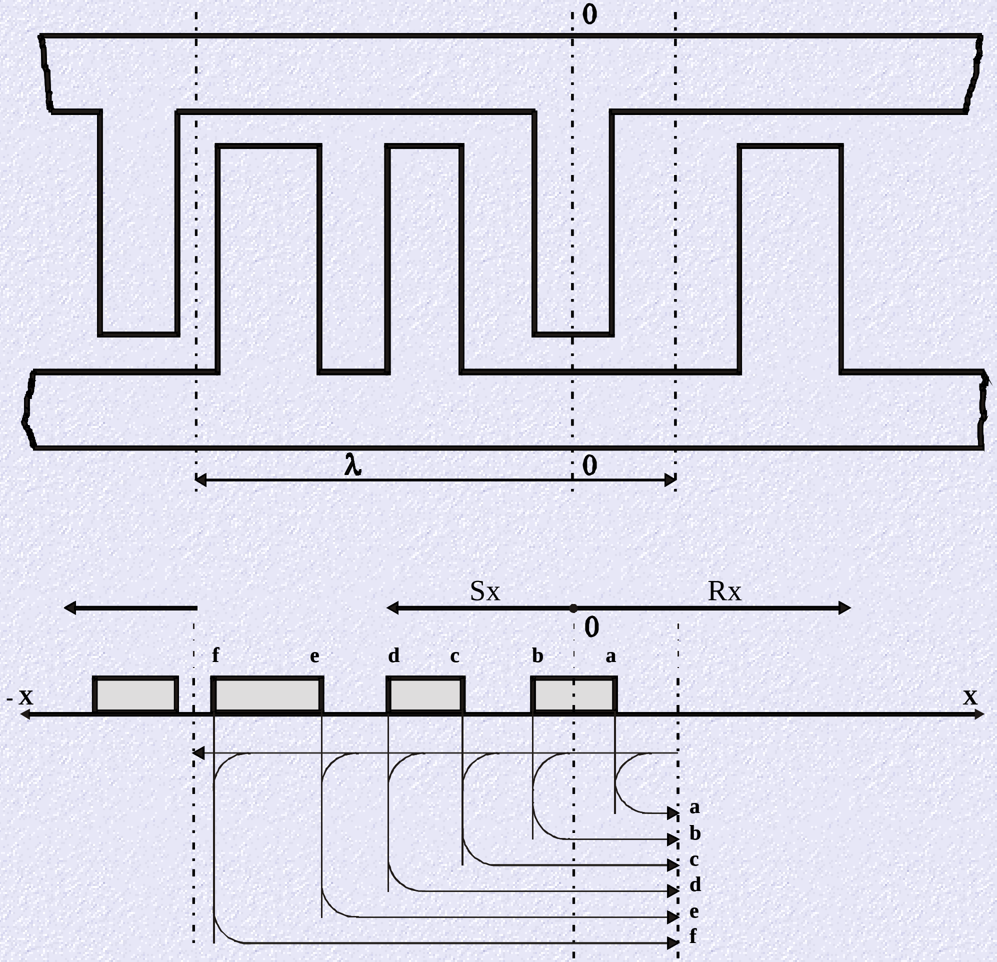


Single phase unidirectional transducers (SPUDT)









Forward amplitude

$R_x = E[(1\text{-}r)\text{-}j\ 0{,}73\ r]$.

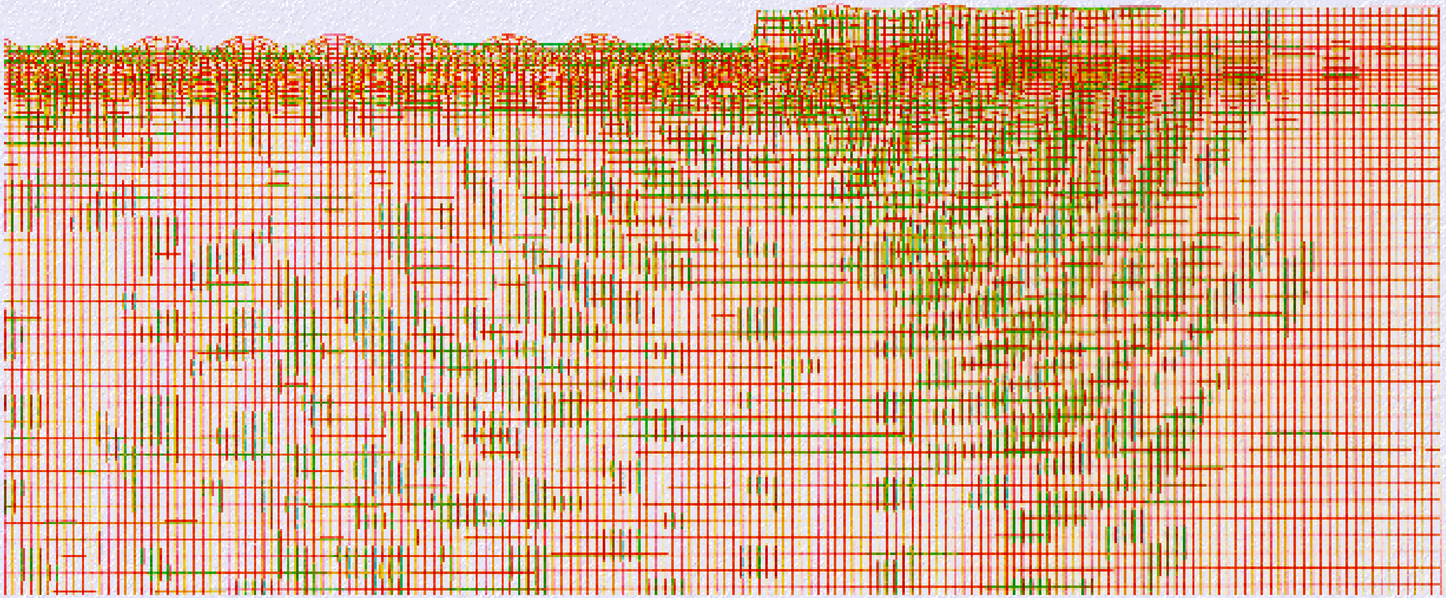
Backward amplitude

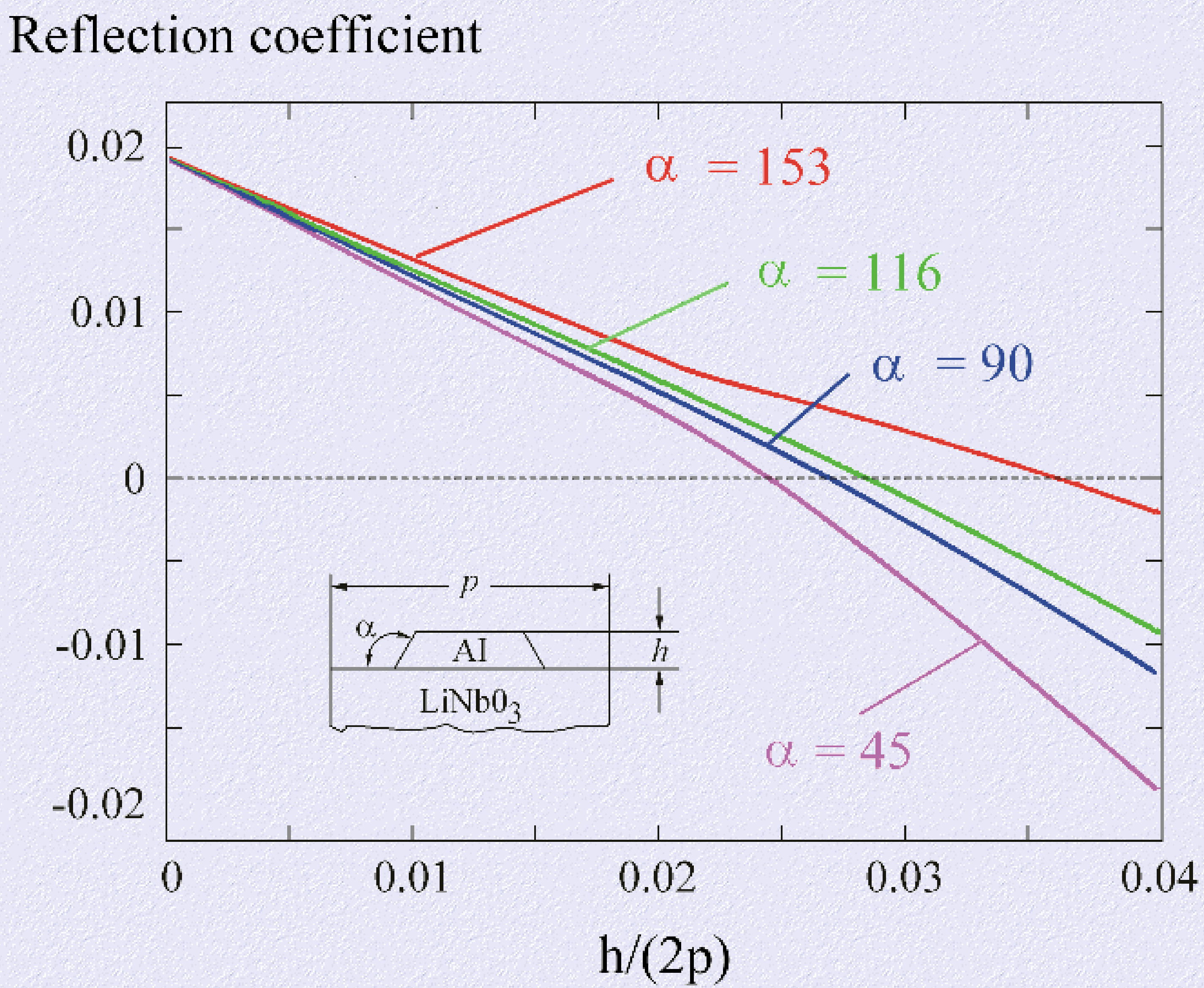
$S_x = E[(1+r)\text{-}j\ 0{,}73\ r]$,

E – electric field
r – refl. Coeff from edge of the strip

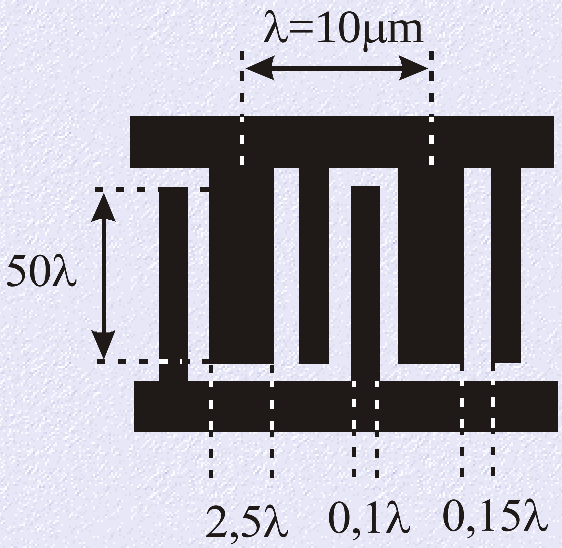
This method ensure almost zero-attenuation inside the band-pass but not very big out of the band

Simulation of the reflection of strip edge



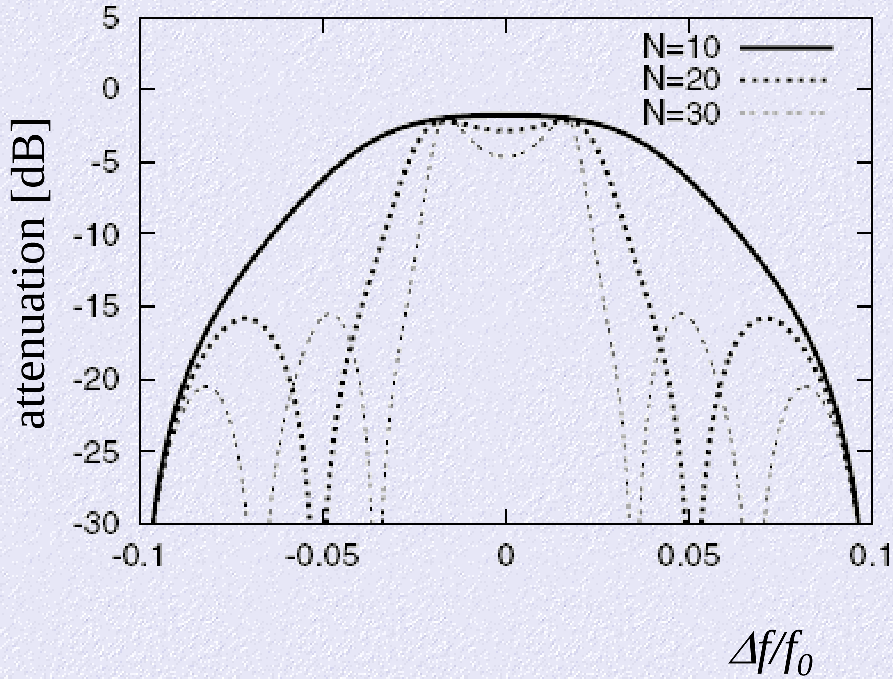

Reflection coefficient
0.02
0.01
0
-0.01
-0.02
0
0.01
0.02
0.03
0.04
h/(2p)
α = 153
α = 116
α = 90
α = 45
p
α
Al
h
LiNb0₃

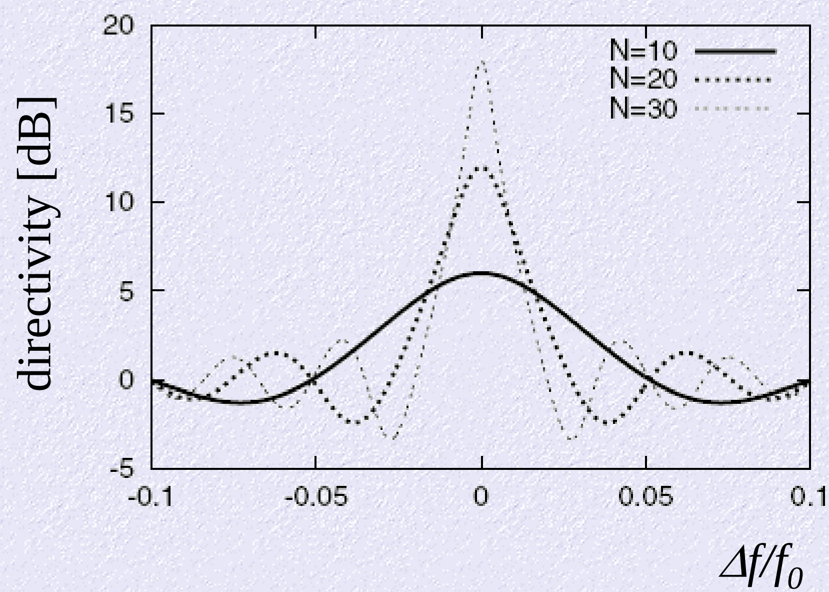
One section of SPUDT



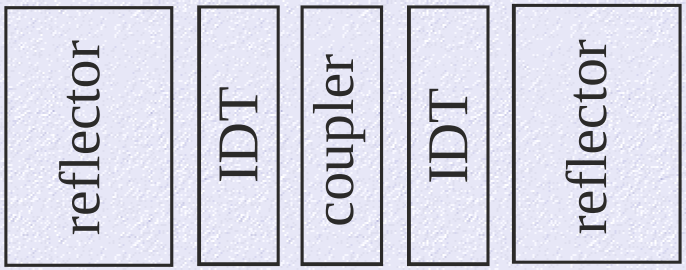
350 MHz

128° XY $LiNbO_3$

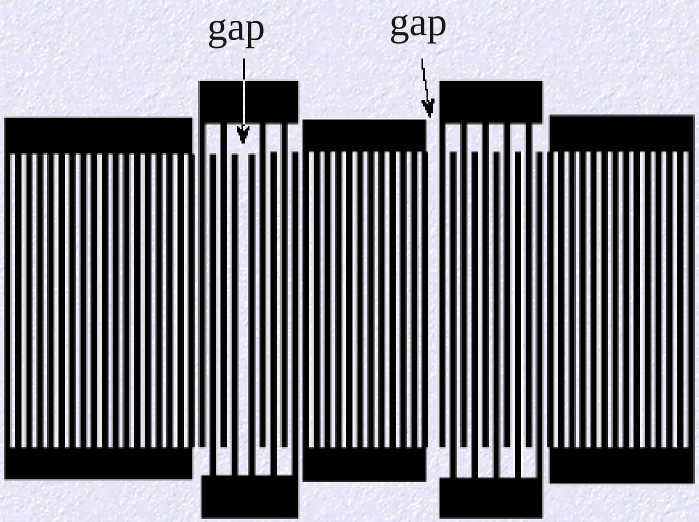




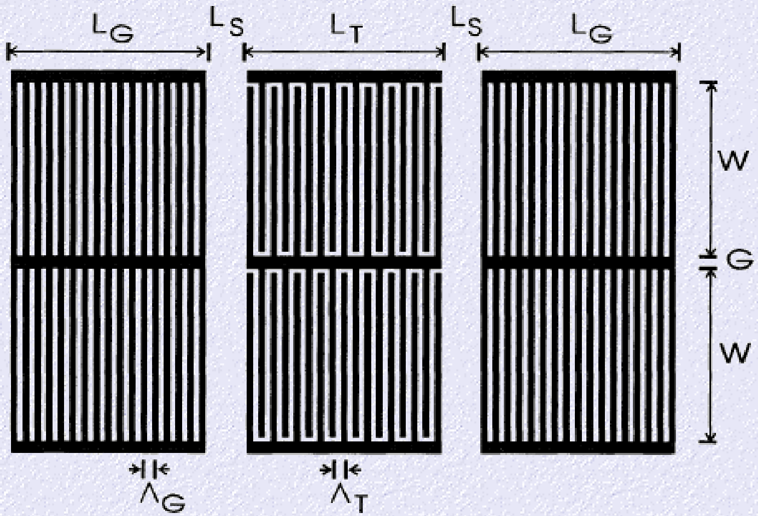
Resonsnt filters

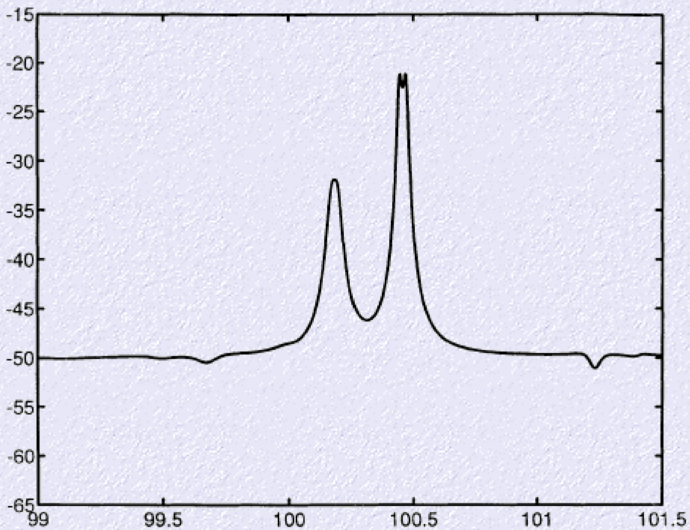


Basic principle



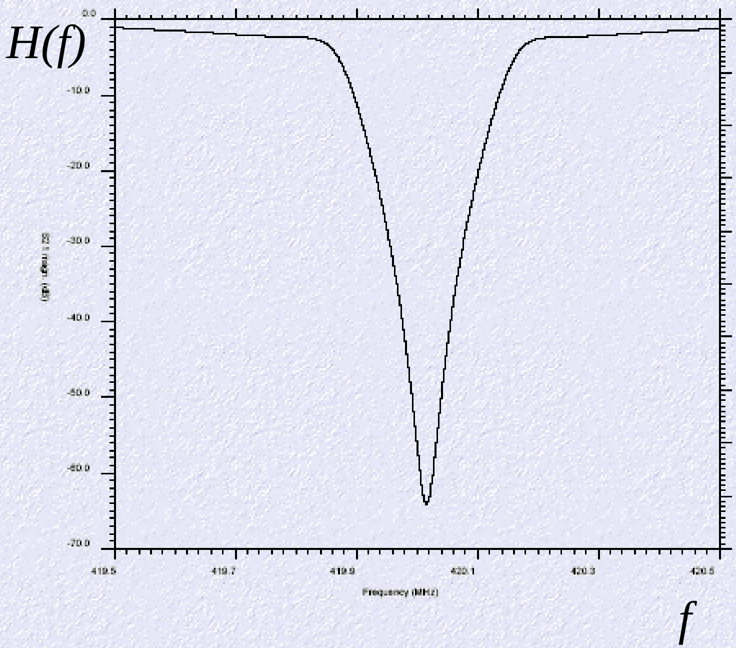
exapmle

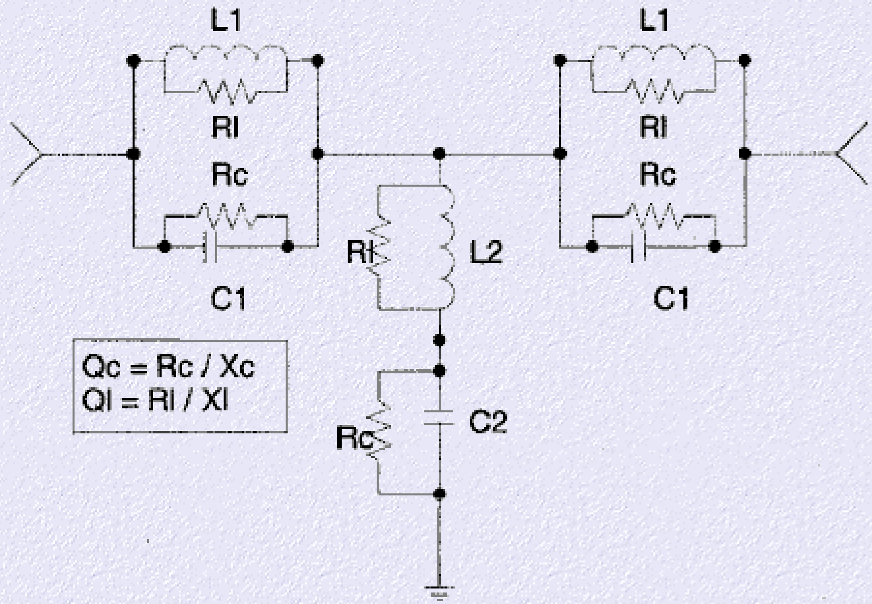


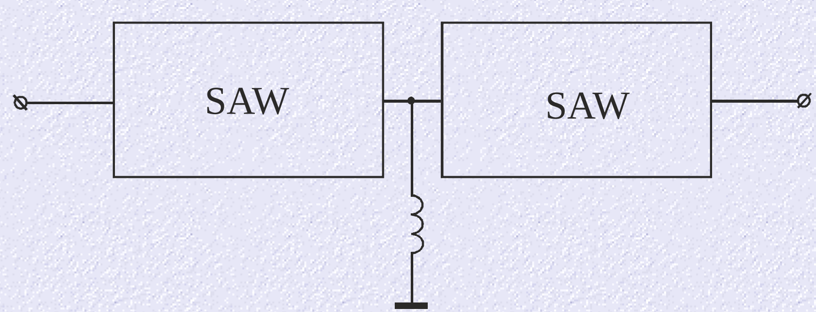


Transversally coupled filter

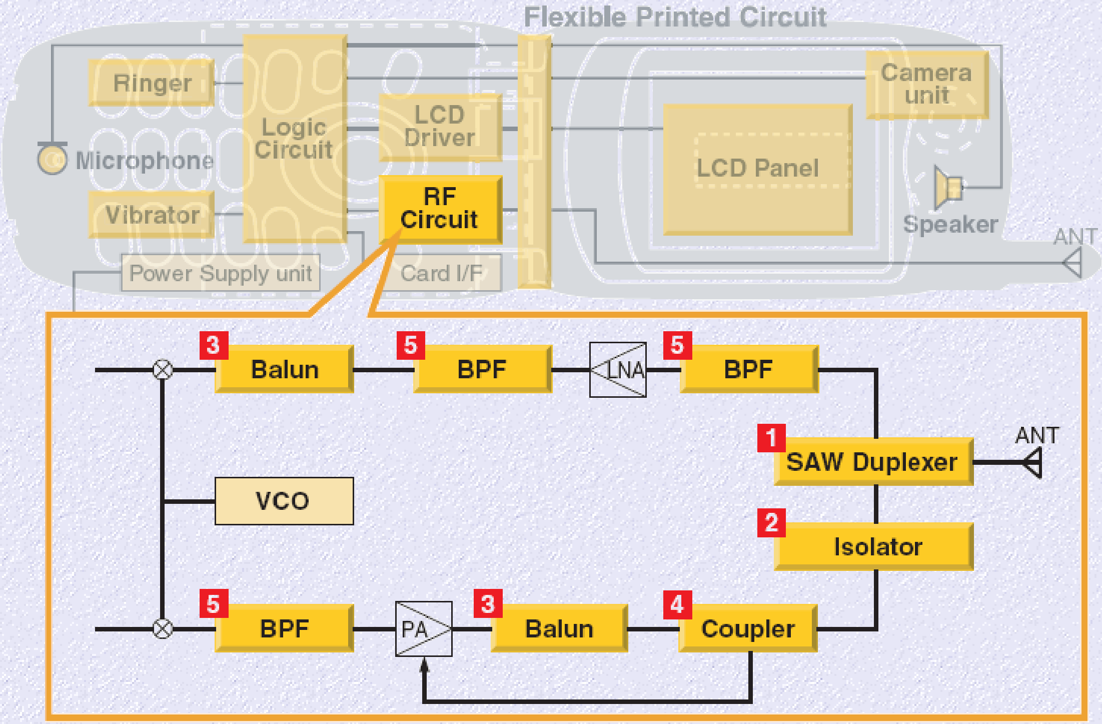
Notch filters

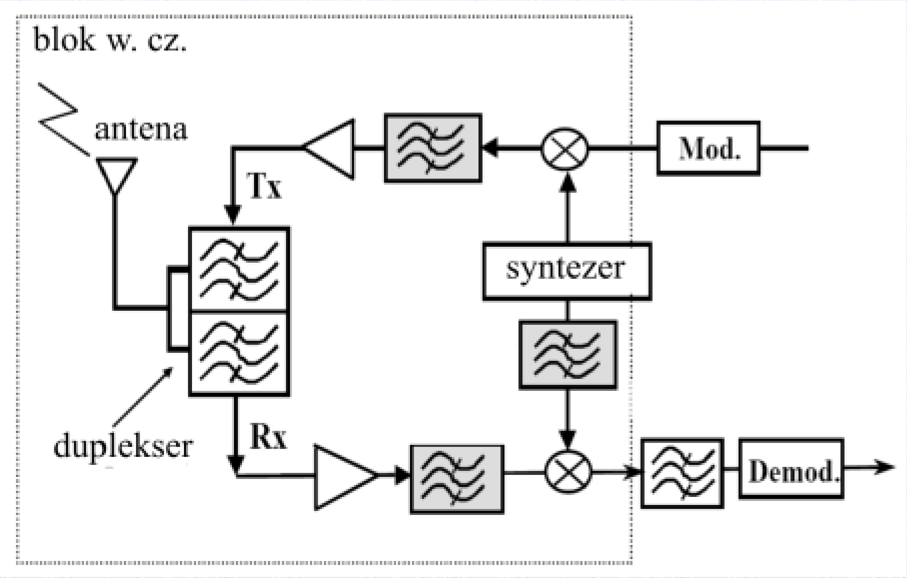


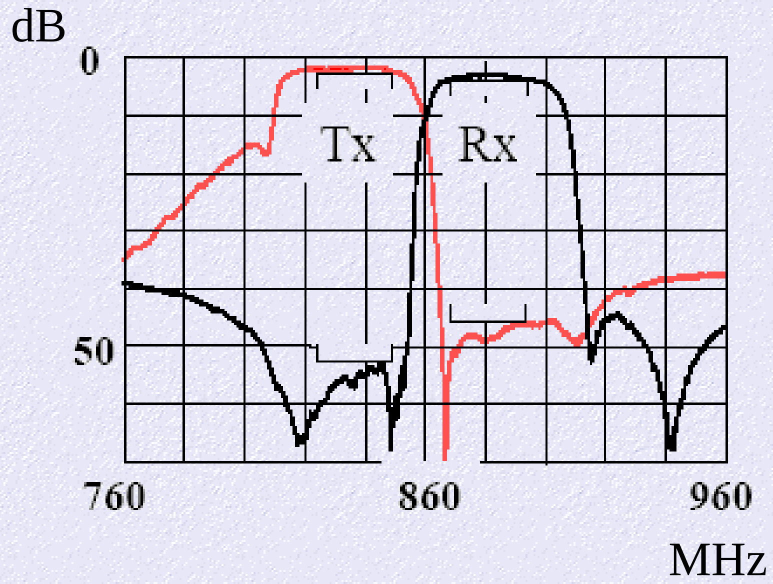




Antenna duplexers







Typical parameters

attenuation

Tx 1,2 dB

Rx 1,8 dB

suppresion

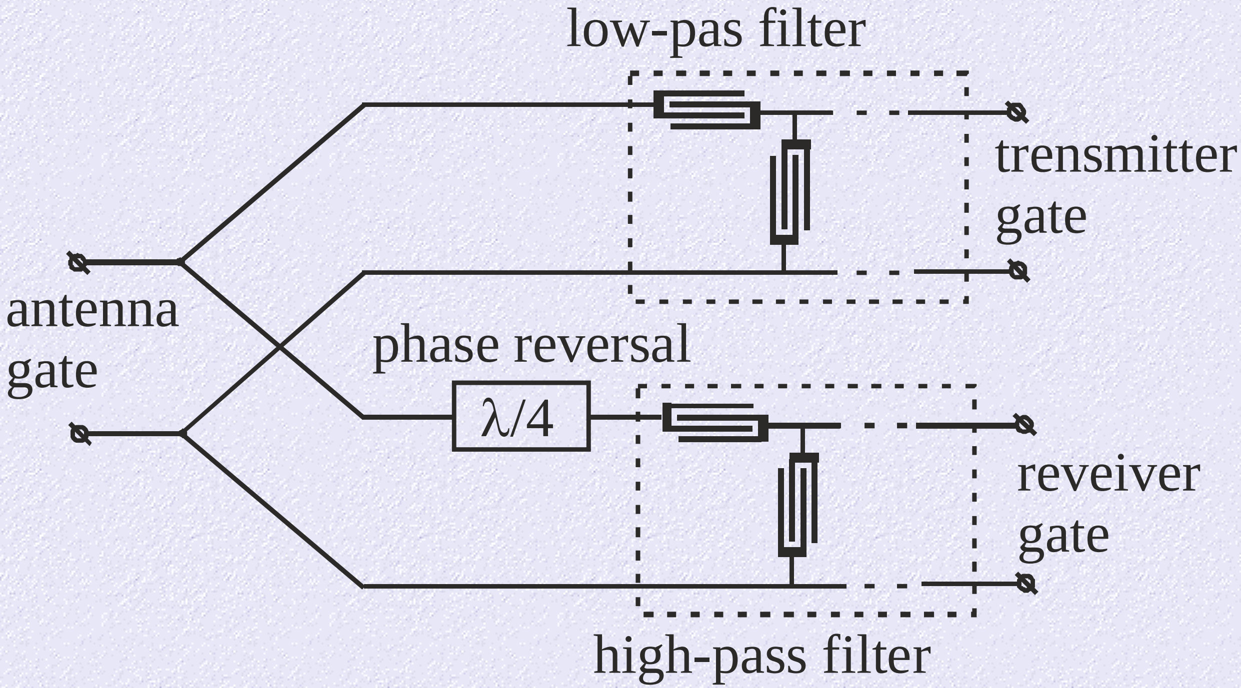
Tx 53 dB

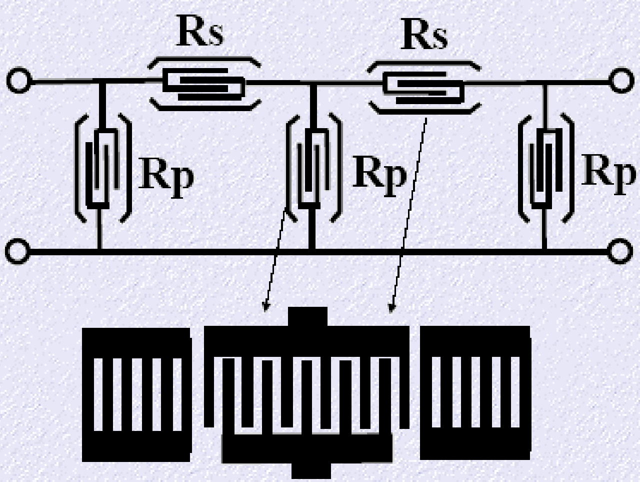
Rx 45 dB

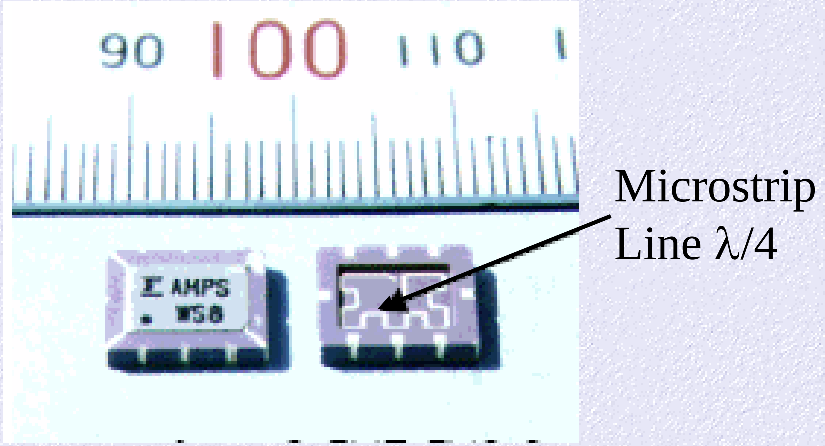
SWR

Tx 1,6

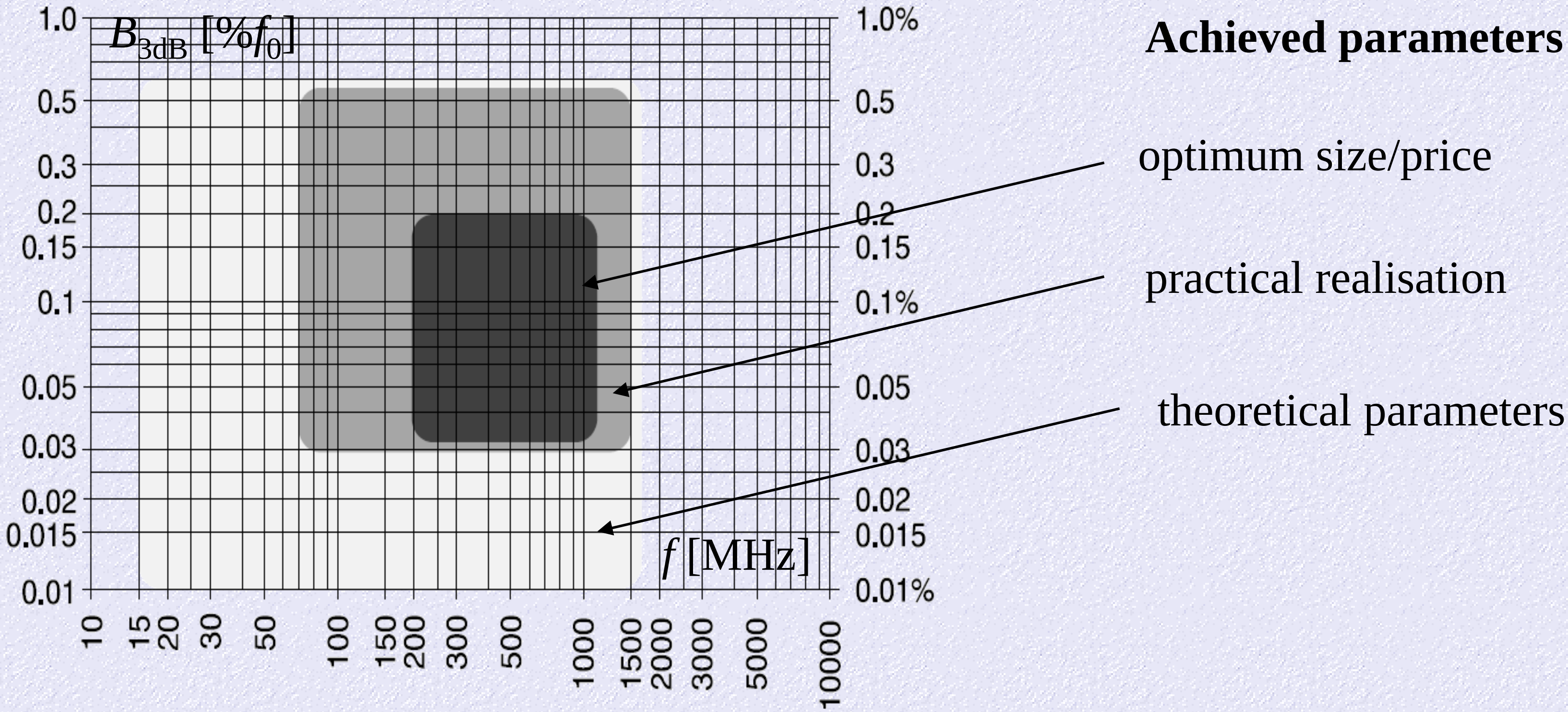
Rx 1,7

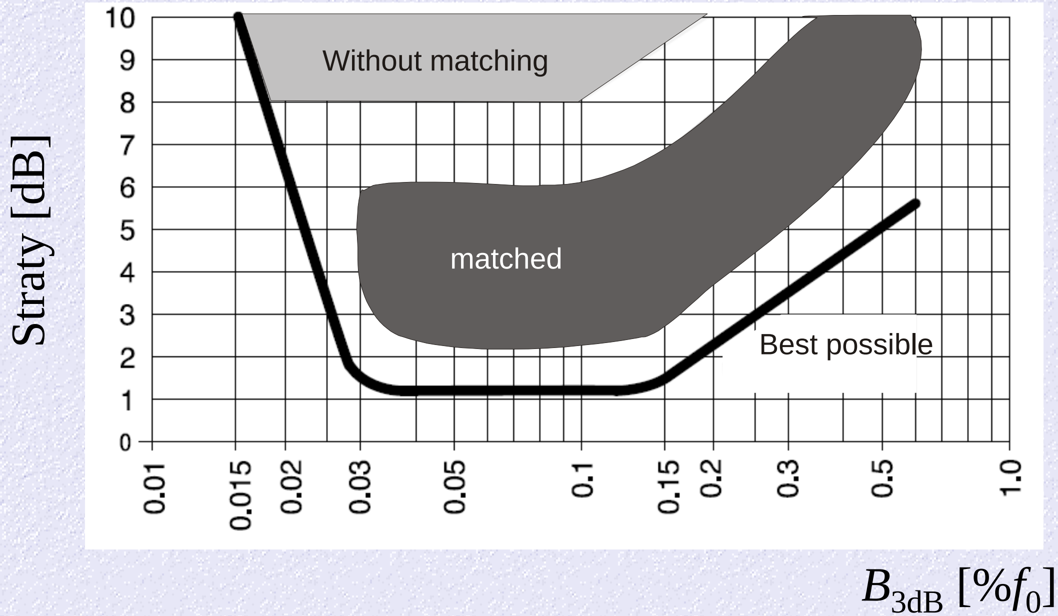




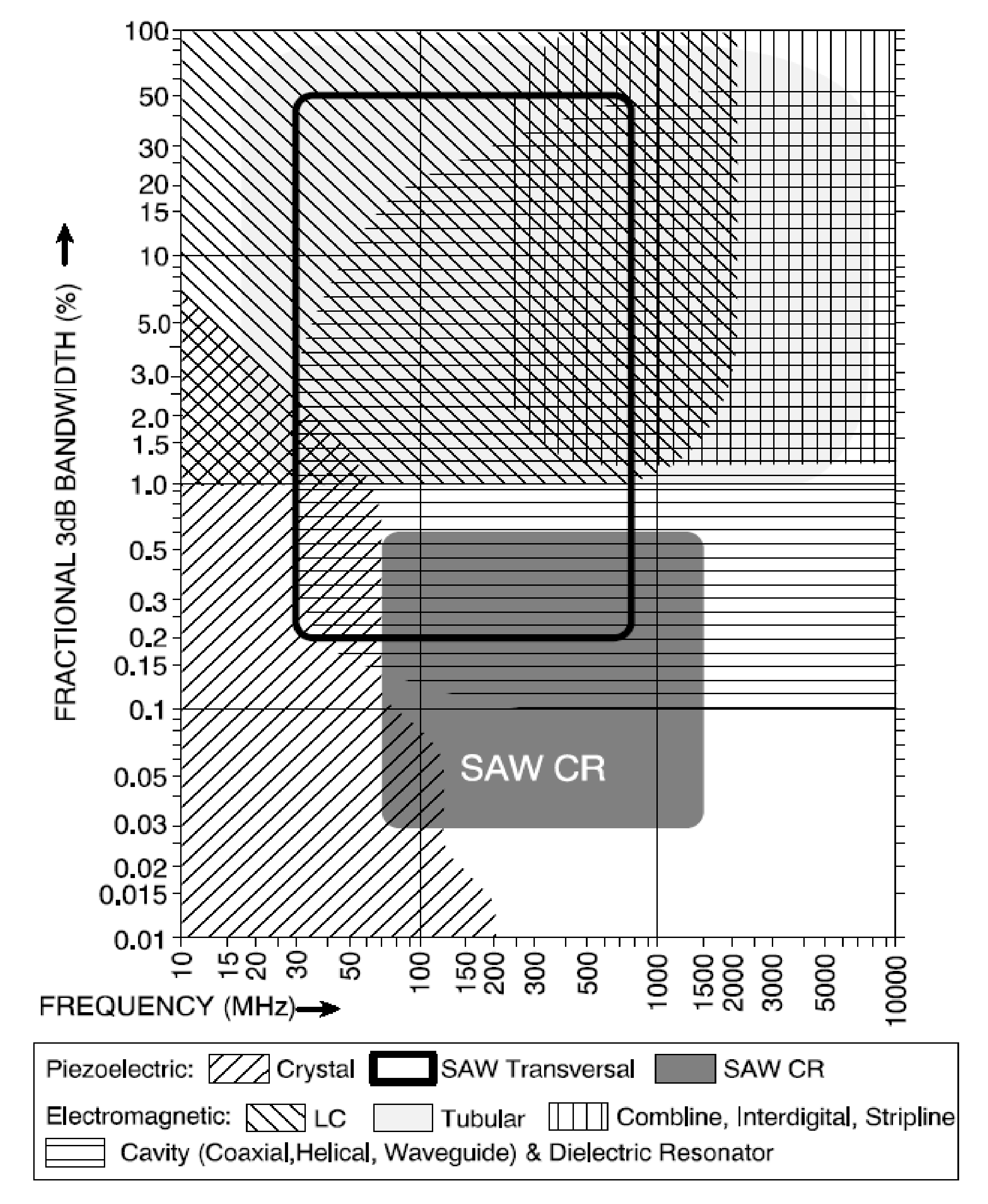


Achieved parameters

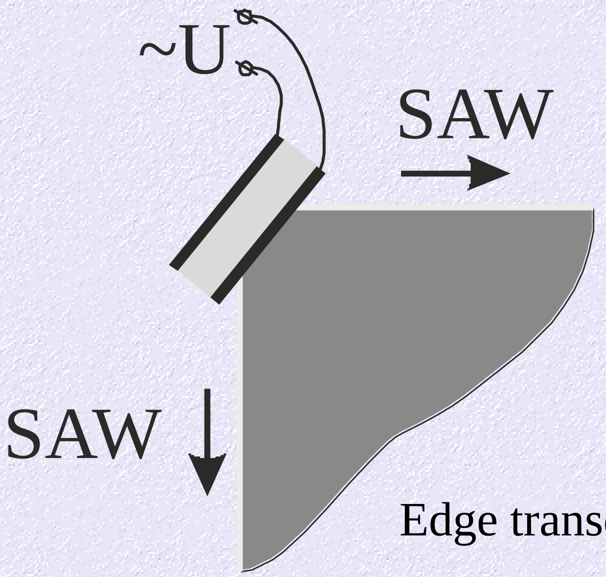




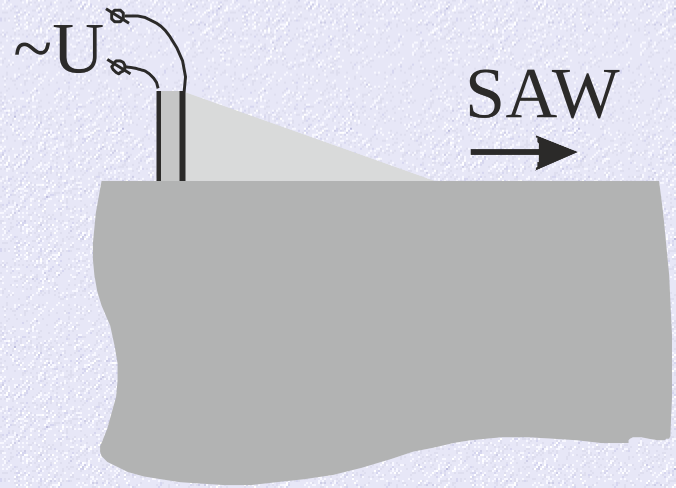
Different technology comparizon



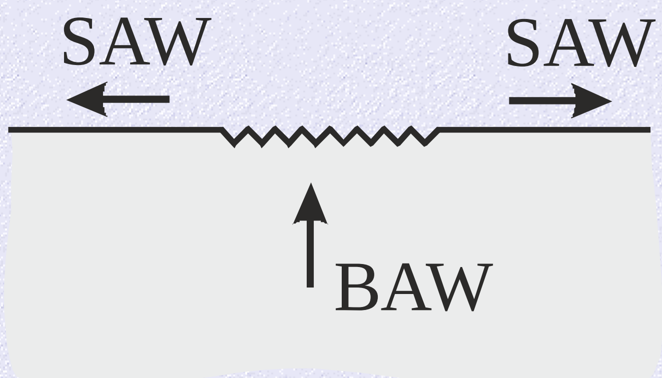
Other methods of SAW excitation



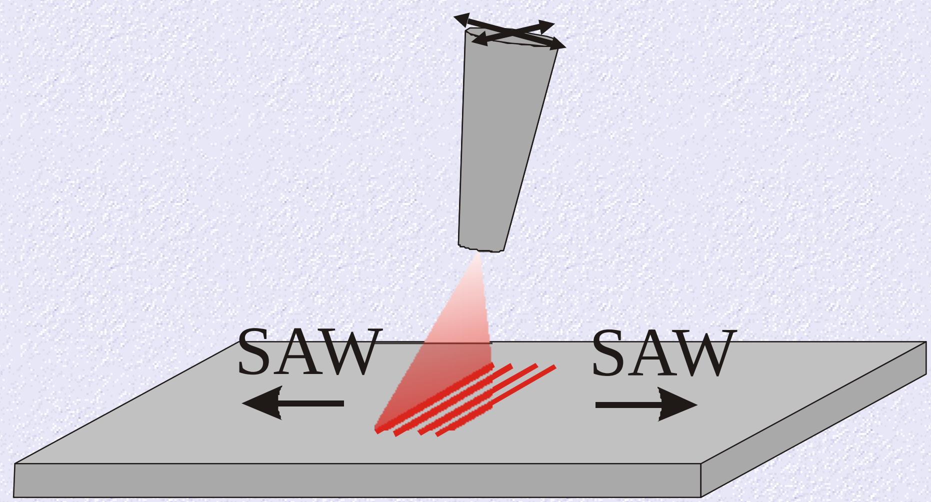
Edge transducer



Chock transducer



grating transducer



Laser transducer

IDT modelling

δ function model takes into consideration only general properties of IDT
It ignores e.g.

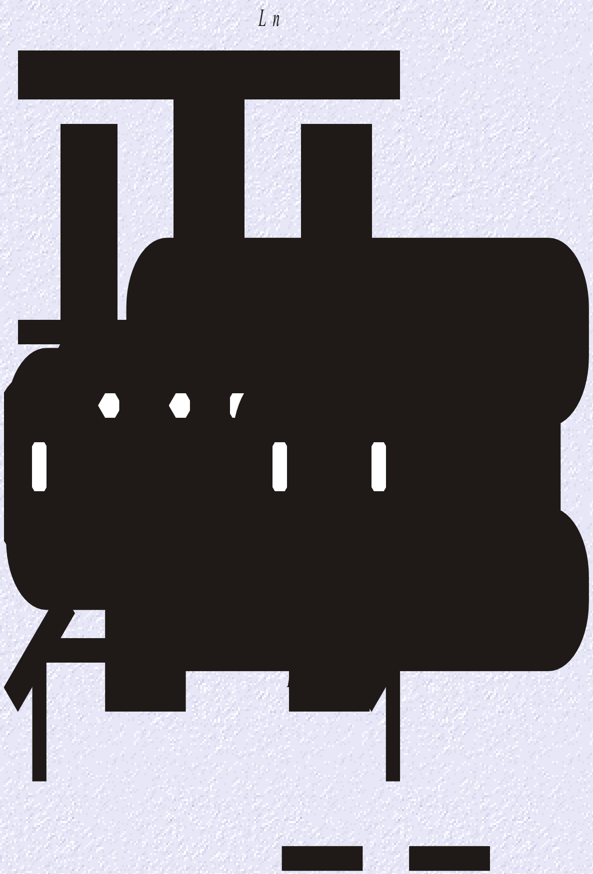
SAW reflections:
- interelectrodes,
- inter IDT transducers (odd echoes)
- from structure border

Diffraction effects
BAW generation and reflection
SAW leakage
SAW velecity change due to mass or electrical loading

To take above into consideration other models are often applied:

- Enhanced δ
- Equivalent circuit method
- Spectrum method
- Matrix (Y, S),
- Coupled of modes (COM)
- Ramez algorithm
- Numerical (FEM i BEM)
- others

Equivalent circuit



$$\begin{bmatrix} D_{n+1} \\ e_{n+1} \end{bmatrix} = \begin{bmatrix} \cos\alpha_n & \sin\alpha_n \\ -\sin\alpha_n & \cos\alpha_n \end{bmatrix} \begin{bmatrix} D_n \\ e_n \end{bmatrix} + E_n \begin{bmatrix} -\sin\alpha_n \\ 1-\cos\alpha_n \end{bmatrix}$$

One IDT section

$$Z = jZ_0 tg\frac{\alpha_n}{2} \qquad \alpha_n = \frac{\pi f}{2 f_n}$$

$$f_n = \frac{v}{2L_n} \qquad Z_1 = \frac{jZ_0}{\sin\alpha_n}$$

Acoustic part

$$D_n = jZ_o i_n$$

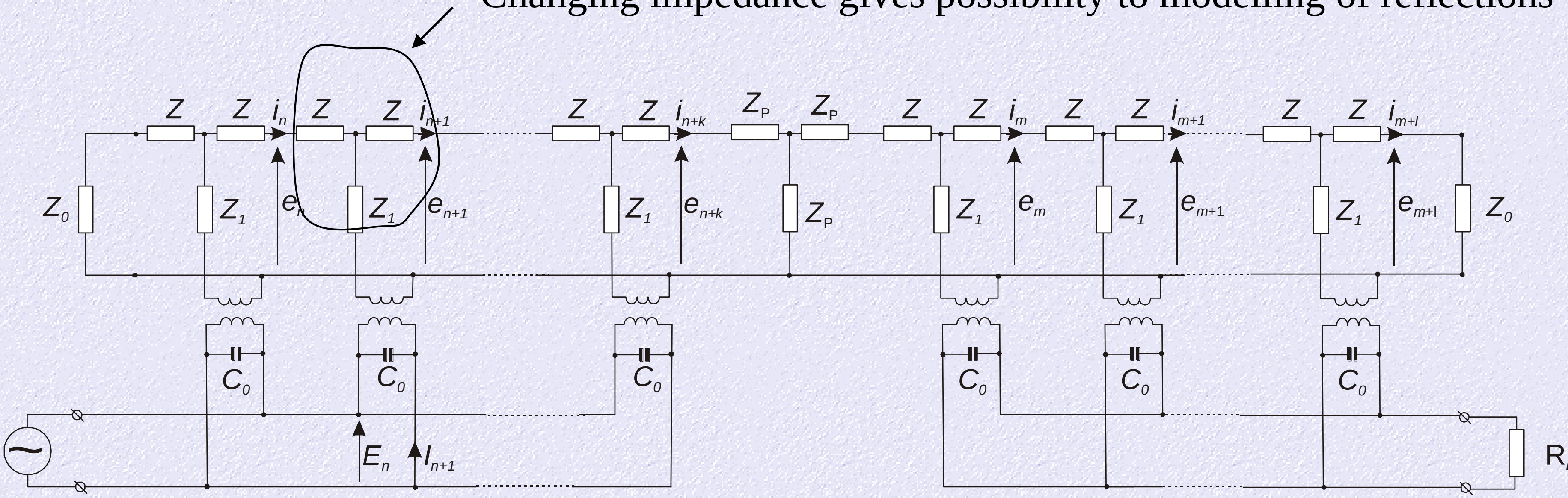
$$I_n = j\omega C_0 E_n + \frac{j(D_n - D_{n+1})}{Z_0}$$

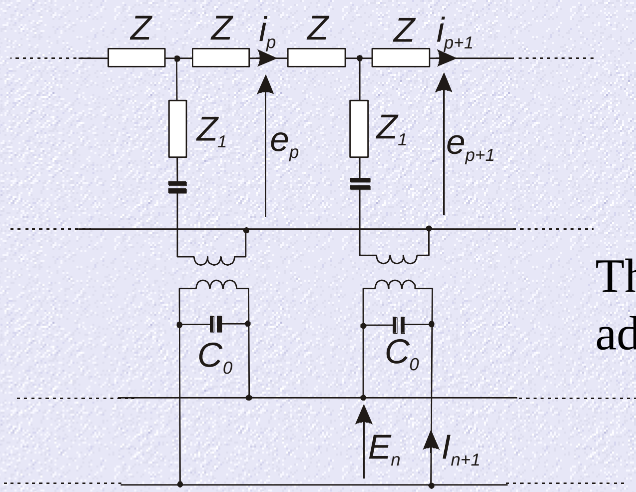
Electric part

$$Z_0 = \frac{2\pi}{\omega_0 C_0 K_s^2}$$

Two collaborating IDTs

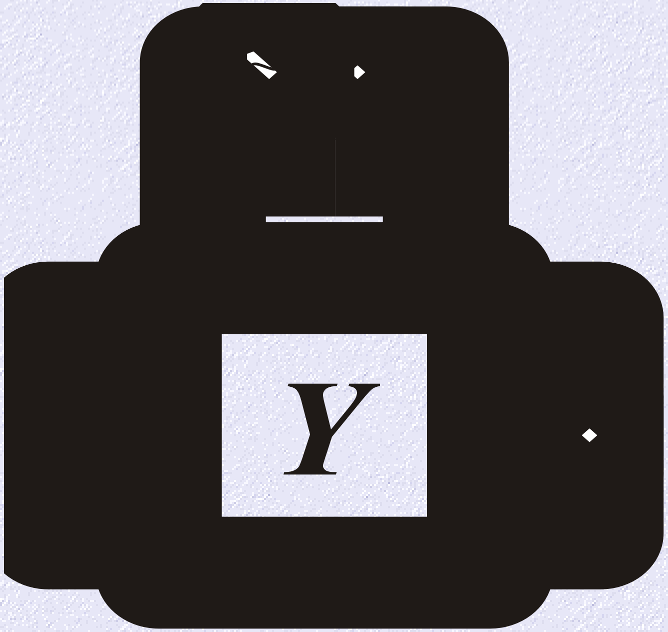
Changing impedance gives possibility to modelling of reflections





The reflections can be taken into consiferation adding capacitances

Admitance matrix model



Acoutic port

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{12} & Y_{11} & -Y_{13} \\ Y_{13} & -Y_{13} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Electric port

Electric port

According electric circuit theory the three-port network can be described by following set:

$$\begin{cases} \begin{bmatrix} I_N \\ I_0 \end{bmatrix} = [Y] \begin{bmatrix} U_N \\ U_0 \end{bmatrix} \\ E_g = I_N R_g + U_N \\ U_0 = -I_0 R_0 \end{cases}$$

Independent components of the matrix $[Y]$ are definied using the following relations

$$[Y] = \begin{bmatrix} y_{nn} & y_{n0} \\ y_{0n} & y_{00} \end{bmatrix}$$

$$y_{nn} = \frac{I_N}{U_N}\Big|_{U_0=0} \qquad y_{n0} = \frac{I_N}{U_0}\Big|_{U_N=0}$$

$$y_{0n} = \frac{I_0}{U_N}\Big|_{U_0=0} \qquad y_{00} = \frac{I_0}{U_0}\Big|_{U_N=0}$$

$$y_{n0} = y_{0n}$$

The solution of the equations set allows to obtain transfer function:

$$H(\omega)=\frac{U_0}{E_g}=\frac{y_{on}R_0}{(1+y_{nn}R_g)(1+y_{00}R_0)-y_{0n}^2R_0R_g}$$

Ignoring three-pass echo $y^2{}_{0n}R_0R_g$ one can obtain simpler form

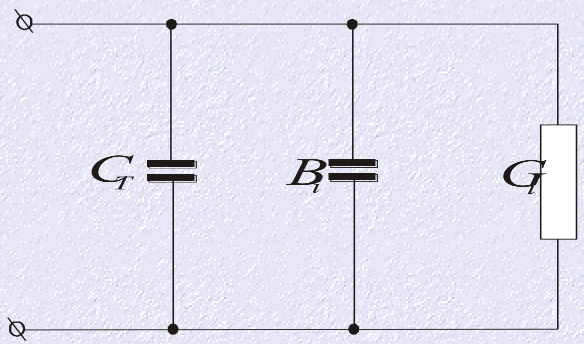
$$H(\omega)=\frac{H_\delta(\omega)R_0}{(1+y_{nn}R_g)(1+y_{00}R_0)}$$

Input and output admittance

$$y_{ii}(j\omega)=G_i(\omega)+jB_i(\omega)+j\omega C_T$$

The admitance $y_{ii}(j\omega) = G_i(\omega) + jB_i(\omega) + j\omega C_T$

Can be represented by equivalent circuit



$$C_T = W\sum_{n=1}^{N}\sum_{m=1}^{N} b_n b_m Q_{m-n}$$

C_T is static capacity of IDT with electrodes having aperture W

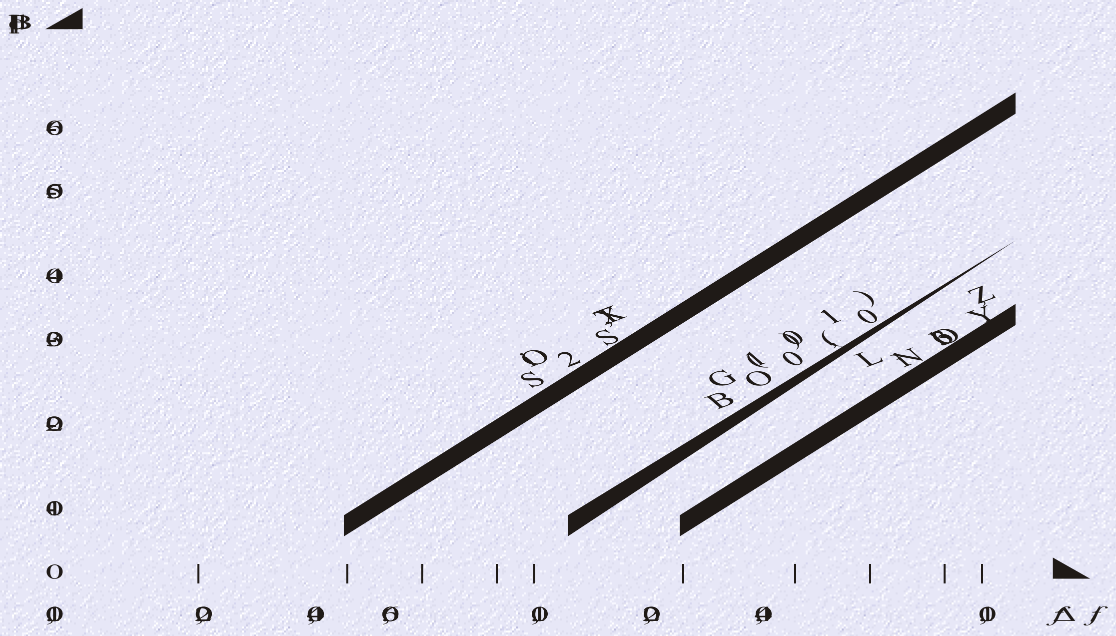
Susceptance B and conductance G are linked by Hilbert transfirm:

$$B_i(\omega) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{G_i(\omega')}{\omega - \omega'}\, d\omega'$$

Insertion loss:

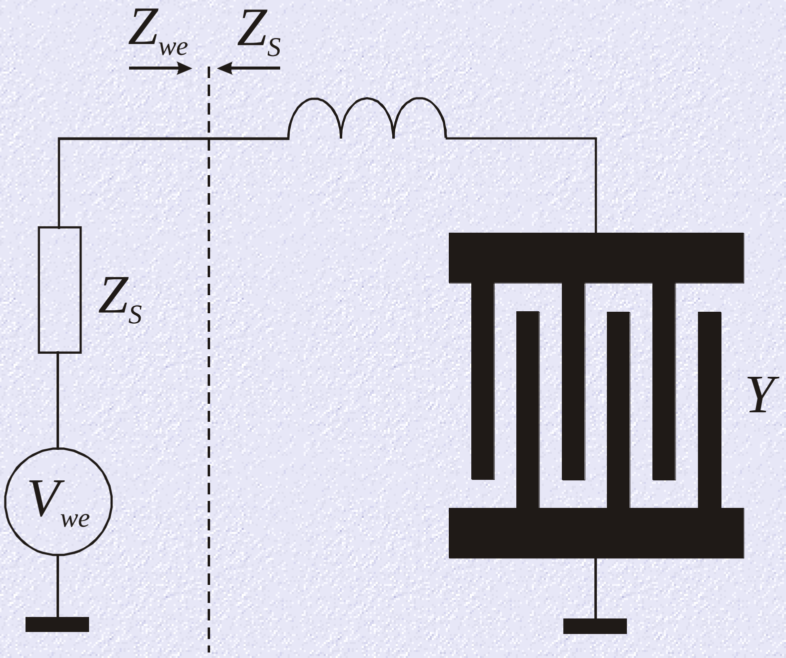
$$ST = -10\log\frac{P_0}{P_g} = -10\log\left(\frac{4R_g\left|H(\omega)\right|^2}{R_0}\right)$$

Design calculations



Minimal insertion losses depend on IDT bandwidth

Parameters of IDT on quartz ST?



$$v \approx 3150 \ \text{[m/s]}$$

$$\varepsilon_{11} = \varepsilon_{22} = 4,51\varepsilon_0$$

$$\varepsilon_0 = 8,8542 \cdot 10^{-12} \left[\frac{C^2}{Nm^2}\right]$$

$$\frac{\Delta v}{v} \approx 0,001$$

$$\varepsilon_{33} = 4,6\varepsilon_0$$

$$\varepsilon_{13} = 0$$

$$\varepsilon_{ef} = \varepsilon_0 \sqrt{\varepsilon_{11}\varepsilon_{33} - \varepsilon_{13}^2} \approx 4,55\varepsilon_0$$

Maximal bandwitdh is obtain when acoustic and electric Q-factors are egual

$$Q_a = N_p = Q_e = \frac{\omega C}{G_a} = \frac{\omega(\varepsilon_0 + \varepsilon_{ef})WN_p}{\omega(\varepsilon_0 + \varepsilon_{ef})W\frac{\Delta v}{v}N_p^2\tilde{G}} = \frac{1}{\frac{\Delta v}{v}N_p^2\tilde{G}}$$

For simple (non-splitted electrodes) $\tilde{G} = 2,87$

Number of pairs of electrodes $N_p = \sqrt{\frac{1}{\frac{\Delta v}{v}\tilde{G}}} \approx\sim 18,7 \to 19$

3 dB bandwidth $\left.\frac{\Delta\omega}{\omega}\right|_{3dB} \approx \frac{1}{N_p} \approx 5,2\%$

IDT impedance for center frequancy

$$Z = \frac{1}{\frac{1}{R_a} + j\omega C} = \frac{R_a}{1 + j\omega CR_a} = \frac{R_a\left(1 - j\omega CR_a\right)}{1 + \left(\omega CR_a\right)^2} = \frac{R_a}{1 + \left(\omega CR_a\right)^2} - j\frac{\omega CR_a}{1 + \left(\omega CR_a\right)^2}$$

IDT capacity have to be compensated by inductivity - on center frequency:

$$\frac{R_a}{1+\left(\omega C R_a\right)^2} = \frac{1}{G_a + \dfrac{\omega^2 C^2}{G_a}} = R_0$$

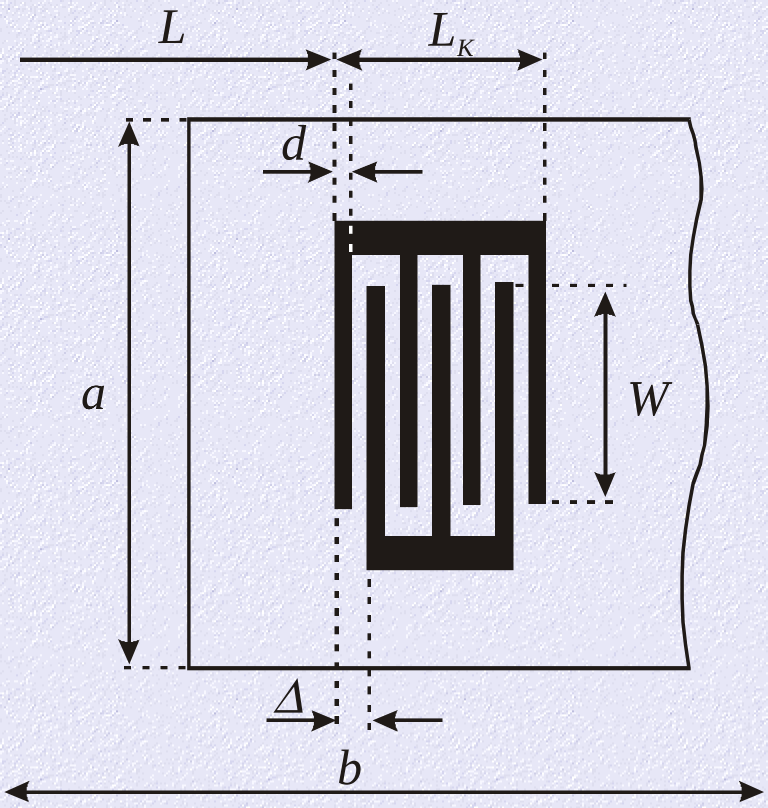
assuming $G_a << \omega C$ i.e. $\dfrac{1}{R_a} << \omega C$ we obtain

$$\frac{1}{G_a + \dfrac{\omega^2 C^2}{G_a}} = \frac{G_a}{\omega^2 C^2} = \frac{2\pi v\left(\varepsilon_0 + \varepsilon_{ef}\right)\dfrac{W}{\lambda}\dfrac{\Delta v}{v}N_p^2 \tilde{G}}{\left[2\pi v\left(\varepsilon_0 + \varepsilon_{ef}\right)\dfrac{W}{\lambda}\dfrac{\Delta v}{v}N_p\right]^2} = \frac{\dfrac{\Delta v}{v}\tilde{G}}{2\pi v\left(\varepsilon_0 + \varepsilon_{ef}\right)\dfrac{W}{\lambda}} = R_0$$

so

$$\frac{W}{\lambda} = \frac{\dfrac{\Delta v}{v}\tilde{G}}{R_0 2\pi v\left(\varepsilon_0 + \varepsilon_{ef}\right)} \approx 59$$

Simple filter (identical IDTs)



Number of digits in IDT

$$N = 2\alpha f_0 / \Delta F$$

$$\alpha = 0,6 \div 0,8$$

Optimal digids number for giving substrate

$$N_{opt} = \sqrt{\pi / k_m^2}$$

Podłoże	v_s [km/s]	k_m^2	ε_r
Kwarc	3.15 – 3.2	0.0012 – 0.0024	4.52–4.55
$LiNbO_3$	3.5 – 4.0	0.005 – 0.058	25–60
$Bi_{12}GeO_{20}$	1.62 – 1.7	0.007 – 0.0164	38–45
$Bi_{12}SiO_{20}$	1.7	0.018	
$LiTaO_3$	3.2 – 3.4	0.0069 – 0.0093	43–51

Optimum deviation

$$P = \left(N_{opt}/N\right)^2$$

period of structure – electrodes λ/4

$$\Delta = \nu_s / 2 f_0$$

$$d = \Delta/2$$

minimal aperture

$$W_{\min} = \sqrt{L\lambda_s}$$

L inter IDT distance

IDT length $$L_k = N\Delta - \Delta/2$$

Minimal substrate dimensions $$a = W + 2(\Delta + l) \qquad b = L + 2(L_k + l)$$

l distance of IDT from end of the substrate

$$B_1 = -10\lg\left[1\Big/\left(1+P\right)^2\right]$$ Reflection coefficient

$$B_2 = -10\lg\left[P^2\Big/\left(1+P\right)^2\right]$$ Tranmission coefficient

$$B_3 = -10\lg\left[2P\Big/\left(1+P\right)^2\right]$$ IDT insertion loss

Filter loss $B = 2B_3$

Distortion level due to reflections $B_d = 2B_1$

IDT capacitance

$$C_0 = NC_1 W/2$$

$$C_1 \cong 2\left(1+\varepsilon_r\right)\left(6.5s^2 + 1.08s + 2.37\right)$$

$$s = d/\Delta$$

Matching inductance

$$L = 1 / 4\pi f_0^2 C_0$$

Radiation resistance

$$R_0 = R_r(f_0) = 2k_m^2 / \pi^2 f_0 C_1 W$$

Transfer function

$$K_F(j\omega) = \frac{K_1(j\omega)}{K_1(j\omega_0)} K_2(j\omega) K_3(j\omega) \frac{K_4(j\omega)}{K_4(j\omega_0)}$$

Transfer function of input circuits

$$K_1(j\omega) = \frac{Z}{R + j\omega L + Z}$$

$$Z = R_a(f) + jX_a(f) + 1/j\omega C_0 \qquad R_a(f) = R_0(\sin X / X)^2$$

$$X_a(f) = R_0(\sin 2X - 2X)/2X^2 \qquad X = \pi N(f - f_0)/2f_0$$

IDTs transmittances

$$K_2(j\omega) = K_3(j\omega) = \frac{\sin X}{X}$$

Input admittance

$$K_4(j\omega) = R/(R + Z + j\omega L)$$

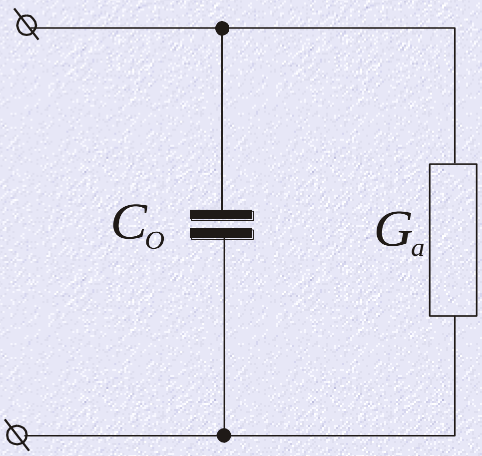
Total ransmittance

$$K_\Sigma(j\omega) = K_F(j\omega) + B_d \exp(-2\omega t_{\nu L})$$

Total time delay $$t_{\nu L} = (L + L_K)/\nu_p$$

Some matching remarks

$$Z_{IDT} = \frac{1}{(1/R_a) + j\omega C_0} = \frac{R_a}{1+(\omega C R_a)^2} - j\frac{\omega C R_a}{1+(\omega C R_a)^2}$$



Necessary to compensate

$$\frac{R_a}{1+(\omega C R_a)^2} = \frac{1}{\frac{1}{R_a} + R_a(\omega C)^2} = \frac{G_a}{G_a^2 + (\omega C)^2} = R_{\dot{Z}R}$$

$$\frac{G_a}{G_a^2 + \left(\omega C\right)^2} = R_{ŹR}$$

$$G_a = 2\pi v(\varepsilon_0 + \varepsilon_{ef})\frac{W}{\lambda}\frac{\Delta v}{v}N_p^2\tilde{G}$$

for IDT

$$C = (\varepsilon_0 + \varepsilon_{ef})WN_p$$

so
$$\frac{\frac{\Delta v}{v}\tilde{G}}{2\pi v(\varepsilon_0 + \varepsilon_{ef})\frac{W}{\lambda}} = R_{ŹR}$$

Matching process starts together with design process.

Example of simple IDT on 36° YX $LiTaO_3$ for 50Ω source

$$\left(\tilde{G} = 2,87\right)$$

v = 4109 [m/s]
$\Delta v/v$ = 2,8 [%]
$\varepsilon_{11} = 40,90\varepsilon_0$
$\varepsilon_{33} = 41,45\varepsilon_0$
$\varepsilon_{13} = 0$
$\varepsilon_0 = 8,854 \cdot 10^{-12}$

$$\frac{\frac{\Delta v}{v}\tilde{G}}{2\pi v(\varepsilon_0 + \varepsilon_{ef})\frac{W}{\lambda}} = R_{\acute{Z}R}$$

$$\frac{\frac{\Delta v}{v}\tilde{G}}{R_{\acute{Z}R}\,2\pi v(\varepsilon_0 + \varepsilon_{ef})} = \frac{W}{\lambda}$$

$$\varepsilon_{ef} = \sqrt{\varepsilon_{11}\varepsilon_{33} - \varepsilon_{13}^2} = 41,18\varepsilon_0$$

$$\frac{W}{\lambda} = \frac{\frac{\Delta v}{v}\tilde{G}}{R_{\acute{Z}R}\,2\pi v(\varepsilon_0 + \varepsilon_{ef})} = \frac{0.028 \cdot 2,87}{50 \cdot 2\pi \cdot 4109 \cdot \left(1 + 41,48\right) \cdot 8,854 \cdot 10^{-12}} \approx 167$$